Summary of Research on Experiences Intended to Deepen Teachers' Mathematics Content Knowledge

Studies of two types of experiences were included in the review of research related to deepening teachers' mathematics content knowledge. First, nine studies investigated the effects of teachers' experience in professional development programs that had deepening teachers' mathematics content knowledge as a goal. Second, five studies examined teaching practice as a context for teachers to deepen their mathematics content knowledge. One study fit into both categories, yielding a total of thirteen studies in the review.

If you are interested in how these studies were selected and reviewed, a summary of the methodology can be found at
http://www.mspkmd.net/index.php?page=02_4a-3c2

Effects of Programs Aimed at Deepening Teachers' Mathematics Content Knowledge

Studies of the effects of nine different interventions designed to deepen teachers' mathematics content knowledge were reviewed. Information about the research studies is displayed in Table 1. Information about the interventions examined in the eight studies is shown in Table 2.

In all nine studies, participating teachers' mathematics content knowledge increased. At a minimum, these results provide existence proofs that experiences aimed at deepening teachers' mathematics content knowledge can achieve this goal. It is important, however, to bear in mind that studies with positive effects are probably more likely to be submitted, and accepted, for publication than those with no effects.

The diversity of the programs investigated across these nine studies suggests that there are a variety of effective ways of structuring and delivering experiences to deepen teachers' mathematics content knowledge. The programs also differed in the grade range of participating teachers and the mathematics content strands that were addressed. Positive effects were found for experiences with teachers from elementary, middle, and secondary grades variously targeting algebra; data analysis, probability and statistics; geometry; measurement; and number and operations; as well as problem solving and representation. On the whole, more empirical evidence exists regarding interventions for middle grades[1] and elementary[2] teachers than for secondary teachers.[3]

In some cases, the interventions were described in detail,[4] which is helpful for understanding teachers' experiences and interpreting the link between the intervention and the effects on

[1] Basista & Mathews, 2002; Clark & Schorr, 2000; Garner-Gilchrist, 1993; Geer, 2001; Sowder, Phillip, Armstrong, & Schappelle, 1998; Swafford, Jones, & Thornton, 1997; Swafford Jones, Thornton, Stump, & Miller, 1999.

[2] Basista & Mathews, 2002; Franke, Carpenter, Fennema, Ansell, & Behrend, 1998; Garner-Gilchrist, 1993; Geer, 2001; Hill & Ball, 2004; Stecher & Mitchell, 1995; Swafford et al., 1997; Swafford et al., 1999.

[3] Basista & Mathews, 2002; Geer, 2001.

[4] Franke et al., 1998; Sowder et al., 1998; Stecher & Mitchell, 1995.

teachers' mathematics content knowledge. In several cases, however, the intervention was described only partially,[5] making it more difficult to support these interpretations.

All of the programs consisted of either a long-term course or an intensive workshop lasting at least two days. Two of the programs studied included semester-long courses,[6] four of the studies included intensive summer workshops lasting at least a week,[7] and three of the studies included workshops held during the school year that were shorter than a week.[8] These courses or workshops focused on a specific topic in mathematics and situated teachers' conceptual learning within the work they do in classrooms. Half of the studies included follow-up sessions or seminars in addition to the courses or workshops,[9] some of the studies included observations and interviews to support the application of what the teachers learned.[10]

All nine of the programs studied were fairly extensive, requiring at least one week of commitment (typically more) and multiple meetings; the teachers were indicated to be volunteers in nearly all cases.[11] In at least two cases, the teachers were also screened prior to selection for participation in the interventions to ensure that they were committed to changing their teaching practice.[12] Generalizability of the findings from these studies must be considered in light of these parameters, because the populations that these teachers represent are limited to teachers willing and able to commit to participation in such extensive interventions.

Eight of the programs that were studied included attention to both disciplinary content knowledge and pedagogical content knowledge, although in varying degrees of emphasis.[13] One of these programs also addressed ways of knowing in mathematics,[14] and the remaining program addressed both teachers' knowledge of ways of knowing in mathematics and their pedagogical

[5] Basista & Mathews, 2002; Clark & Schorr, 2000; Garner-Gilchrist, 1993; Geer, 2001; Hill & Ball, 2004; Swafford et al., 1997; Swafford et al., 1999.

[6] Clark & Schorr, 2000; Garner-Gilchrist, 1993.

[7] Basista & Mathews, 2002; Geer, 2001; Hill & Ball, 2004; Stecher & Mitchell, 1995; Swafford et al., 1997; Swafford et al., 1999.

[8] Franke et al., 1998; Sowder et al., 1998; Stecher & Mitchell, 1995.

[9] Basista & Mathews, 2002; Franke et al., 1998; Geer, 2001; Hill & Ball, 2004; Swafford et al., 1997; Swafford et al., 1999.

[10] Basista & Mathews, 2002; Franke et al., 1998; Sowder et al., 1998.

[11] Clark & Schorr, 2000; Franke et al., 1998; Garner-Gilchrist, 1993; Geer, 2001; Sowder et al., 1998; Swafford et al., 1997; Swafford et al., 1999.

[12] Garner-Gilchrist, 1993; Sowder et al., 1998.

[13] Basista & Mathews, 2002; Clark & Schorr, 2000; Empson, 1999; Featherstone et al., 1995; Hill & Ball, 2004; Sowder et al., 1998; Stecher & Mitchell, 1995.

[14] Featherstone et al., 1995.

content knowledge.[15] Although for most studies, unfortunately, it was not clear what level or combinations of levels of disciplinary content knowledge was addressed, it was evident that across the studies the level of disciplinary content knowledge addressed varied among student-level content ideas, more advanced disciplinary content, and a more profound understanding of fundamental mathematics ideas. In addition, the programs attended to different aspects of knowledge of ways of knowing in mathematics and pedagogical content knowledge. It is not possible from this small set of studies, with varying goals for deepening teachers' content knowledge, to know what kinds of programs are the most efficient or effective for achieving particular goals. One study[16] did examine variations in teachers' experiences of different professional development workshops in relation to their content knowledge gains. These analyses suggested that summer institutes of greater duration, and those that focused on mathematical analysis, reasoning, and communication had larger impacts on teachers' mathematics content knowledge. The researchers advised caution with respect to these results due to the fact that approximately one-fourth of the eligible institutes agreed to participation in the study, so biases in the samples of professional development experiences and teachers could have affected the findings.

The Evidentiary Base for Claims about Programs Aimed at Deepening Teachers' Mathematics Content Knowledge

It is important to recognize that particular features of the programs, although described in detail in some cases and logically tied to the reported impacts on teachers' mathematics content knowledge, were not investigated in any of the studies through systematic or naturalistic variation. Findings in these studies can only be understood to result from teachers' experience of the programs as a whole.

Different measures were used across the studies, and the programs had intended impacts on teachers' mathematics content knowledge that were not measured. As a result, it is not possible to identify whether features of one program may be more or less effective for a particular purpose than features of another program. Claims that some features are important for deepening teachers' mathematics content knowledge are suggested to some extent by their presence in the multiple programs studied. The importance of these features in deepening particular facets of teachers' content knowledge was supported on logical or theoretical grounds in all of the studies. One study examined two program features empirically and found positive relationships between both longer program duration and greater mathematical emphasis on teachers' content knowledge gains.[17] However, the contributions of particular features to effects on different aspects of teachers' content knowledge cannot be strongly concluded from the empirical evidence in these studies.

[15] Swafford et al., 1997; Swafford et al., 1999.

[16] Hill & Ball, 2004.

[17] Hill & Ball, 2004.

Another important consideration for interpreting the results of several of the studies was delivery of the interventions by the researchers,[18] which in some cases were also the developers of the interventions. When researchers deliver interventions, it increases the likelihood that program descriptions are accurate. When they develop and deliver, it is more likely that interventions are delivered as intended. However, these researchers, whether developers or deliverers, have a vested interest in study outcomes, potentially introducing biases toward evidence of intended outcomes. Also, implementation of the programs may have included aspects that remained implicit and would therefore not appear in researchers' descriptions, making replication of the interventions very difficult.

Although all of these studies but one[19] used either a pre-post design to measure changes in teachers' content knowledge or traced changes in teachers' content knowledge over time, none of these studies used comparison groups of teachers who did not participate in the professional development programs. Given the experience levels of many of the participating teachers, the extent of professional development provided, and the nature of the measured changes, it is certainly reasonable to argue that the changes resulted from the interventions, but without comparisons to other teachers these claims are not solidly grounded in

[18] Basista & Mathews, 2002; Clark & Schorr, 2000; Franke et al., 1998; Sowder et al., 1998; Swafford et al., 1997; Swafford et al., 1999.

[19] Stecher & Mitchell, 1995.

Table 1
Studies of Interventions to Deepen Teachers' Mathematics Content Knowledge: Study Characteristics

Name of Study	Purpose of study		TCK Data types		Knowledge Outcomes			Measures of Teacher Content Knowledge				Measurement description		
	Program evaluation	Providing examples	Quantitative	Qualitative	Disciplinary Content	Ways of Knowing	Pedagogical Content	Assessments	Interviews	Observations	Other approach	Validity	Reliability	Triangulation
Integrated Science and Mathematics Professional Development Programs (Basista & Mathews, 2002)	●		●	●	●		●	●		●				
Teachers' Evolving Models of the Underlying Concepts of Rational Number (Clark & Schorr, 2000)	●			●	●		●	●	●	●				
Understanding Teachers' Self-Sustaining, Generative Change in the Context of Professional Development (Franke et al., 1998)		●		●	●		●		●	●		●	●	●
Mathematics Institute: An Inservice Program for Training Elementary School Teachers (Garner-Gilchrist, 1993)		●	●		●		●	●						
Science and Mathematics Professional Development at a Liberal Arts University: Effects on Content Knowledge, Teacher Confidence and Strategies, and Student Achievement (Geer, 2001)	●		●		●		●	●				●		
Learning mathematics for teaching: Results from California's Mathematics Professional Development Institutes (Hill & Ball, 2004)	●		●		●		●	●				●	●	
Middle-Grade Teachers' Mathematical Knowledge and Its Relationship to Instruction (Sowder et al., 1998)		●		●	●		●	●	●	●		●	●	●
Vermont Teachers' Understanding of Mathematical Problem Solving and "Good" Math Problems (Stecher & Mitchell, 1995)	●			●		●	●	●	●			●	●	●
Increased Knowledge in Geometry and Instructional Practice (Swafford et al., 1997) The Impact on Instructional Practice of a Teacher Change Model (Swafford et al., 1999)	●		●		●	●	●	●[a]				●	●	●

[a] Indicates use of an existing measure that was not developed specifically for the purpose of this study.

Table 2
Studies of Interventions to Deepen Teachers' Mathematics Content Knowledge: Intervention Characteristics

Name of Study	Grade Level	Intervention[a]				Content/Process Strand							
		Full description	Teacher involvement voluntary	STEM faculty involved	Researcher(s) involved	Number and operations	Algebra	Geometry	Measurement	Data, probability, statistics	Communication	Problem solving	Representation
Integrated Science and Mathematics Professional Development Programs (Basista & Mathews, 2002)	4–10	N	?	Y	Y	●	●	●		●	●	●	●
Teachers' Evolving Models of the Underlying Concepts of Rational Number (Clark & Schorr, 2000)	6–8	N	Y	?	Y	●							
Understanding Teachers' Self-Sustaining, Generative Change in the Context of Professional Development (Franke et al., 1998)	1–3	Y[b]	Y	?	Y	●		●					
Mathematics Institute: An Inservice Program for Training Elementary School Teachers (Garner-Gilchrist, 1993)	4–8	N	Y	Y	N		●	●		●			
Science and Mathematics Professional Development at a Liberal Arts University: Effects on Content Knowledge, Teacher Confidence and Strategies, and Student Achievement (Geer, 2001)	4–9	N	Y	Y	N	●	●	●	●	●			
Learning mathematics for teaching: Results from California's Mathematics Professional Development Institutes (Hill & Ball, 2004)	K–6	Y[b]	?	Y	N	●					●		●
Middle-Grade Teachers' Mathematical Knowledge and Its Relationship to Instruction (Sowder et al., 1998)	6–8	Y	Y	Y	Y	●							
Vermont Teachers' Understanding of Mathematical Problem Solving and "Good" Math Problems (Stecher & Mitchell, 1995)	4	N	N	?	N							●	
Increased Knowledge in Geometry and Instructional Practice (Swafford et al., 1997) The Impact on Instructional Practice of a Teacher Change Model (Swafford et al., 1999)	4–8	N	Y	Y	Y		●	●		●			

[a] Y = Yes, N = No, ? = Not clear from document
[b] Includes reference with full description.

empirical evidence. For example, it is possible that the teachers might perform better on a measure of content knowledge on a post-test simply because they had completed it previously, in one case[20] only a few weeks earlier. The use of multiple measures addresses this concern to some extent, as in Swafford and colleagues' study[21] in which the participating teachers performed better in three different content areas, and on three separate measures of knowledge of geometry, following treatment, and in two other studies which used both written instruments and interviews with teachers to measure teacher content knowledge.[22]

Teaching Practice as a Context for Deepening Teachers' Mathematics Content Knowledge

Also included in this set were four studies that investigated whether teachers can deepen their mathematics content knowledge as a result of their teaching practice itself. In all four of these cases, the teachers had been, or were simultaneously, involved in an experience to support their mathematics teaching practice. For this reason, each of these studies also included an intervention, although the intervention may not have been directly focused on deepening teachers' mathematics content knowledge. Table 3 shows information about the research studies, and Table 4 displays information about the interventions examined in these four studies. In addition to these four studies, the research study by Franke and colleagues[23] described in Tables 1 and 2 documented teachers' learning about content from their teaching practice during and following their participation in an intervention.

All five of the studies examining teaching practice as a contributor to deepening teachers' content knowledge documented positive effects. The five studies each investigated a different aspect of teaching practice, suggesting that multiple aspects of practice may serve as potential contributors to content knowledge gains. Four of the studies examined elementary school teachers, together spanning grades 1 through 5, and all of these focused on number and operations.[24] The fifth examined secondary teachers, focusing on algebra.[25] Although the number of studies is small and no studies of middle grades teachers were included, there is at least a suggestion that teacher learning of content from practice is possible at multiple grade levels. It is worth noting that all of the studies focused on teachers' learning about a very familiar strand of mathematics for the grade levels being examined. However, no empirical evidence is available to suggest any differences regarding teachers learning particular mathematics content, or more or less familiar content, from their practice.

The majority of the studies used classroom observations and/or interviews to examine the teaching practices of participating teachers. Three of the studies included meetings with teachers

[20] Basista & Mathews, 2002.

[21] Swafford et al., 1997; Swafford et al., 1999.

[22] Sowder et al, 1998; Stecher & Mitchell, 1995.

[23] Franke et al., 1998.

[24] Empson, 1999; Featherstone et al. 1995; Franke et al., 1998; Lin, 2002.

[25] Miller, 1991.

to discuss teaching practices.[26] Student work from the teachers' classrooms was used in four of the studies as tool to focus on student strategies and thinking.[27]

In all five of these studies that investigated teacher learning from practice, the main outcome of interest was pedagogical content knowledge. Two studies also examined teacher learning of disciplinary content knowledge as an outcome,[28] and one of these also addressed knowledge of ways of knowing in mathematics.[29] In all five studies, at least some positive results were reported for each outcome that was investigated, suggesting that teacher learning from practice may include multiple facets of mathematics content knowledge. However, it is worth noting again studies with positive effects are probably more likely to be submitted, and accepted, for publication than those with no effects.

The Evidentiary Base for Claims about Teaching Practice as a Context for Deepening Teachers' Mathematics Content Knowledge

The main purposes of the five studies of teaching practice were to illustrate and substantiate how teachers can learn mathematics content knowledge through their teaching practice. Each of the five studies involved only a small number of teachers, collected only post-experience data, and did not investigate systematic variations, so claims regarding causation or generalizability can be only weakly supported. The common finding in these studies that teaching practice presents a context in which teachers can learn content suggests, however, that efforts to deepen teachers' content knowledge might expand their impact by attending to the context of teaching practice as a site for learning. By providing appropriate structures, resources, and opportunities to support learning, professional development efforts intended to deepen teachers' mathematics content knowledge might take advantage of teachers' ongoing work in their schools and classrooms to bolster their content learning.

Each of the five studies provided examples from either observations or interviews of teachers, or both, to illustrate teachers' learning from their practice. Because this is a fairly new area of investigation, the illustrations of teacher learning in these exploratory studies are a key contribution to building theory about teacher learning from practice. Two of the studies did not present an analysis of data over time that would clearly support claims of teacher learning, although they did link the post-experience data to the teachers' experiences with particular teaching practices.[30]

A few issues regarding validity and generalizability in these studies should also be noted. In three of the studies, systematic methods of analyses were described that included important elements such as establishing reliability among coders and member checking through post-

[26] Empson, 1999; Franke et al., 1998; Miller, 1991.

[27] Empson, 1999; Franke et al., 1998; Lin, 2002; Miller, 1991.

[28] Empson, 1999; Featherstone et al., 1995.

[29] Featherstone et al., 1995.

[30] Empson, 1999; Miller, 1991.

observation interviews. Their overall study designs were aligned with the exploratory and illustrative nature of the research.[31] Methods for selecting the examples that were presented, or for seeking data that are discrepant with the findings, were not apparent in the other two studies, leaving questions about the completeness of interpretation of the full range of data in these studies.[32] In at least three of the five studies, researcher biases toward particular findings, arising because the researchers conducted interventions with the teachers, may have been present.[33]

As exploratory studies, generalizability was not a primary concern. But it is important to bear in mind that the teachers examined in these studies were committed to programs to support improvement and/or investigation of their practice, and that much of their learning may have derived not only from changes in their practice but also from the opportunities they had to reflect on their practice with colleagues and mathematics educators.

Findings in these five studies, commensurate with the purpose of exploratory research, provide a basis for theorizing about teacher learning from practice, and are intriguing as hypotheses to investigate further. Causality is not strongly established by the empirical evidence. Generalizability is supported by thorough descriptions that can be compared to the readers' own experiences with teachers, but not through systematic or representative sampling from a defined population.

[31] Featherstone et al., 1995; Franke et al., 1998; Lin, 2002.

[32] Empson, 1999; Miller, 1991.

[33] Franke et al., 1998; Lin, 2002; Miller, 1991.

Table 3
Studies of Deepening Teachers' Mathematics Content Knowledge Through Their Instructional Practice:
Study Characteristics

Name of Study	Purpose of study		Data types		Knowledge Outcomes			Measures of Teacher Content Knowledge				Measurement description		
	Program evaluation	Providing examples	Quantitative	Qualitative	Disciplinary Content	Ways of Knowing	Pedagogical Content	Assessments	Interviews	Observations	Other approach	Validity	Reliability	Triangulation
On Enhancing Teachers' Knowledge by Constructing Cases in Classrooms (Lin, 2002)		•		•			•		•	•	•	•	•	•
Considerations of Systemic Change and Teachers' Knowledge of Students' Novel Strategies for Whole-Number Operations (Empson, 1999)		•		•	•		•		•					
Constructing Pedogogical Content Knowledge from Students' Writing in Secondary School (Miller, 1991)		•		•			•		•	•	•			
Expanding the Equation: Learning Mathematics through Teaching in New Ways (Featherstone et al., 1995)		•		•	•	•	•		•	•		•	•	•

Table 4
Studies of Deepening Teachers' Mathematics Content Knowledge Through Their Instructional Practice: Intervention Characteristics

Name of Study	Grade Level	Intervention[a]				Content Strand	
		Full description	Teacher involvement voluntary	STEM faculty involved	Researcher(s) involved	Number and operations	Algebra
On Enhancing Teachers' Knowledge by Constructing Cases in Classrooms (Lin, 2002)	1	Y	?	Y	Y	●	
Considerations of Systemic Change and Teachers' Knowledge of Students' Novel Strategies for Whole-Number Operations (Empson, 1999)	3–5	N	N	N	N	●	
Constructing Pedogogical Content Knowledge from Students' Writing in Secondary School (Miller, 1991)	9–12	N	?	?	Y		●
Expanding the Equation: Learning Mathematics through Teaching in New Ways (Featherstone et al., 1995)	2–3	N	Y	Y	N	●	

[a] Y = Yes, N = No, ? = Not clear from document

Bibliography for Summary of Research on Experiences Intended to Deepen Teachers' Mathematics Content Knowledge

If you are interested in how these studies were selected and reviewed, a summary of the methodology can be found at
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