Appendix E

Grades 9–12 Mathematics

Typical Lessons

Ratings of Lesson Components

Overall Lesson Quality

Typical Lessons

The following lesson descriptions are based on a random sample of 9^{th}–12^{th} grade mathematics lessons.

8^{th}–9^{th} Grade Algebra I: Identifying and Graphing Linear Equations and Non-Linear Functions and Relations

This Algebra I lesson consisted mainly of a review of graphing, vocabulary, and an algorithm for finding the equation of a line given points on the line; and a quiz on functions. The teacher changed the text's approach, using the concept and notation of delta for the slope (i.e., slope = $\Delta y/\Delta x$) because he wanted to prepare these students for going into calculus.

The teacher began the class with a short interactive review of graphing, ordered pairs, quadrants, relations, functions, and inverses. The class discussed four ways to describe a relation: mapping, table, ordered pairs, and a T-chart. The teacher reminded students that some relations are functions, and he drew a picture of a mapping from the domain to the range showing that a function cannot include a "duplication of y's for x." The teacher asked how to tell a function from its graph. The students suggested using the vertical line test but some were unsure when the teacher drew and asked about the graph of $x = 5$.

The teacher asked students to tell him what they knew about linear equations. Students suggested $Ax + By = C$ and the teacher told them that if they can get an equation into that form, it is a linear equation. When the teacher suggested the equation $3x^2 - 7x = 2$, the students told him that this was not a linear equation. The teacher helped students come up with the formula, $y = (\Delta y/\Delta x)\,x$, and led the class through a problem in which they began with a table of x,y-values and ended with the equation in slope-intercept form. A student asked about a problem of similar structure that had been assigned for homework the night before. The teacher told the class that he didn't expect them to get that one because it is a parabola and has to have an x^2 term in the equation.

The teacher put eight quiz problems on the overhead, four questions in which students had to determine if various relations were functions and four on evaluating a function given specific numerical values for x (i.e., given $g(x) = x^2 - 3x + 2$, what are $g(-4)$, $g(2a)$, $g(^1/_3)$ and $g(0.1)$). Students worked independently on the quiz, and then the quizzes were graded in class. Because some of the students were confused about the non-linear functions in the last four quiz problems, the teacher graphed the corresponding parabola and showed that it passes the vertical line test. In the evaluation of $g(2a)$ some students thought that $4a^2 - 6a$ should be combined into a single term; the teacher explained why this was not correct. The teacher gave students their midterm grades and told them to study for the upcoming chapter test.

9^{th} Grade: Integers and Equivalent Expressions of Quantities

The teacher stated that the students in this class, 9^{th} graders who are below average in achievement, would not be likely to persist if presented with a difficult intellectual task; he tries to choose tasks that are easy for them to perform and provides lots of explanations and demonstrations of procedures. The lesson topics, integers and equivalent expressions of quantities, were taught because they were next in the faculty's scope and sequence agreement and in the district-adopted textbook.

The teacher began the class by asking students to get out their homework notebook for him to check. He went desk to desk to check whether students had completed the homework assignment; some had not. Students were asked to work independently on a set of six "Do Now" problems displayed on the board:

1. Write 3 numbers between 5 and 6.
2. Write 3 numbers between –5 and –6.
3. $2\frac{1}{2}$, 2.78, and $2\,{}^{3}/_{5}$ fall between which two integers?
4. ${}^{1}/_{7}$, ${}^{2}/_{8}$, and $\frac{3}{4}$ fall between which two integers?
5. $-{}^{4}/_{5}$, $-{}^{3}/_{7}$, and $-{}^{5}/_{9}$ fall between which two integers?
6. Name 3 words between "cheese" and "chicken."

The teacher demonstrated how to change fractions into decimals and instructed students to use the method he demonstrated to answer problems like #3. After discussing problem #6, the teacher observed that there are a finite number of words between "cheese" and "chicken" but the number of numbers between any two integers is infinite. During the discussion some students were attentive, answering questions that the teacher asked and taking notes, while others were not engaged in the lesson. The teacher ignored off-task behavior as long as the students were not disruptive.

Following the discussion of the "Do Now" problems, the teacher said, "Okay, now I'm going to pass out the punishment." He distributed a worksheet that the students were to do for homework. He directed the students to look at a specific homework problem in which they were asked to compare a pair of fractions. The teacher demonstrated, again, how to convert fractions to decimals. The teacher directed students to work on their homework assignment. Some students began working, while others, chose not to do the assignment. (One student appeared to be sleeping; another just sat and stared into space.) As students worked on homework, the teacher circulated through the room, sometimes stopping to answer a question from students. Near the end of class the teacher told students the difference between the = and ≠ signs and demonstrated shading on a number line. The teacher said, "Please have your homework tomorrow," as students packed up slightly in advance of the bell.

9th Grade Algebra Support: Graphing Algebraic Inequalities

The title of this course is Algebra Support. Students in this class (first and second quartile students based on the state's 8th grade assessment) are 9th graders concurrently enrolled in the Algebra Support class and a regular Algebra I class. This lesson on algebraic inequalities was taught as a review lesson to prepare the Algebra Support students for the district-wide Algebra I, Semester I test that they would be taking the next week. In particular, this lesson taught graphing systems of two, two-variable inequalities (e.g., the system $y \geq -x - 2$ and $x - 2y < 4$) and reviewed graphing of two-variable inequalities (e.g., $x + 2y \geq 4$) and graphing of one-variable inequalities (e.g., $|x + 2| < 3$). The content of the lesson was based on Chapter 6 of the district-adopted textbook and was selected by this teacher in collaboration with the other two Algebra Support teachers and the five Algebra I teachers at the school.

The teacher began the lesson by presenting his solutions to two homework problems (e.g., $y > 2x$). He then asked students to turn to a section, "Graphing Linear Equations," in a handout entitled "Intensive Math Algebra Lab First Semester Exam Review." The teacher modeled the solutions for the first two problems on this sheet and asked students to work independently on the third problem. After the students had worked for a while, the teacher modeled the solution. This procedure was repeated for three more problems.

The teacher displayed six quiz problems on the overhead (e.g., Solve and graph $|x + 2| < 3$; Solve, graph, and shade $y - x < 6$). Students worked independently on these problems; as they finished they turned in their quiz papers and waited quietly for the rest of the class to finish. When all students had finished, the teacher asked specific students to work some problems from the review packet on the board. The teacher helped students at the board get the correct answers, and he explained the students' solutions to the rest of the class. At the end of the class the teacher reminded students to turn in their notebooks during the next class and to turn in their completed exam review packet at the beginning of the exam period the following week.

9^{th}–10^{th} Grade Algebra I: Solving Systems of Linear Equations

The lesson topic, solving systems of linear equations, was part of the Algebra I mathematics content mandated by the state and district curriculum guides, and the district's pacing guide specified how to sequence topics and how much time to spend on each topic. The teacher talked about the importance of using the curriculum guide to make sure all topics tested on the state assessment are covered. The teacher classified the students in the class (9^{th} and 10^{th} graders) as being "average to a little below average" and explained that, because the textbook problems are sometimes too difficult for her students, she has to begin with easier problems that she selects from other textbooks. The lesson was the beginning of a unit on systems of equations and focused on finding solutions to systems of equations by graphing; in the subsequent two days the teacher planned to teach students the substitution and elimination methods.

The teacher began the lesson with a whole-class discussion and demonstration of procedures. She asked students to define the terms "infinite" and "coincide" and explained that, in this lesson, they were going to learn how to solve a system of linear equations involving only two equations. The teacher worked three problems (e.g., the system y = 2x and x + y = 3) on the board while students worked the examples at their desks. For the first example the teacher stated steps that the students were to do (e.g., "draw a coordinate plane on your graph paper") and explained her thinking to the class (e.g., when graphing y = 2x, "I'm thinking, well, it's in the y-equals form... y = mx + b... I do not see a 'b' ... So that tells me the y-intercept is 0"). The first system had one solution (i.e., lines intersected at one point), the second had no solution (i.e., the lines were parallel), and the third had an infinite number of solutions (i.e., the two lines coincided). The teacher classified the three systems as consistent independent, inconsistent, and consistent dependent systems, respectively. The teacher asked the students what they saw as drawbacks to the graphing method of solving systems of linear equations. Students were asked to complete another example, a consistent independent system, as independent practice and to share their solutions with the class. The teacher presented, one at a time, three contextual line graphs showing data about two fictitious companies regarding productivity (intersecting lines), production cost (parallel lines), and sales (equivalent lines). She discussed each graph with the class and then asked the class to vote for the company they would hire based on the graphs. The teacher assigned homework, and students worked on the homework assignment until the end of the class period.

9^{th}–11^{th} Grade Algebra I: Solving Linear Inequalities

The teacher explained that his district's Algebra I course of study and the designated Algebra I textbook are aligned by content and that he uses both in the planning of each lesson. In particular, this lesson was designed as a review lesson on the textbook chapter entitled "Solving Linear Inequalities." The teacher said the purpose for this lesson was to help students prepare for the next day's chapter test. Procedures for solving one-variable compound inequalities and absolute value inequalities were the major focus of the lesson.

Most of the class period was used for a whole-class discussion of the homework problems assigned during the previous class. The homework problems required students to solve compound inequalities (e.g., $2 < x + 2 \leq 5$) and to identify properties used in solving this kind of inequality (e.g., If $4x - 1 < 7$, then $4x - 4 < 4$ is an example of the subtraction property for inequality). The assignment consisted of a mixture of multiple choice and open-ended items. The teacher called on students, individually, to give answers to the multiple choice problems, and then he asked students to display their solutions to a few of the open-ended problems on the board. The teacher corrected any errors in the students' board work and answered questions the students asked about solution steps.

The teacher briefly reviewed the meaning of absolute value. He put the problem, $|4k + 2| \leq 14$, and the first solution step, $4k + 2 \leq 14$ and $-1\ (4k + 2) \leq 14$, on the board. Students were given three minutes to complete the problem, working independently. A student was asked to put the complete solution on the board, and the teacher went over some of the steps, but not all, due to time considerations. The teacher concluded the class by assigning a set of homework problems; the assignment provided students with additional practice in solving and graphing compound inequalities and absolute value inequalities.

10^{th} Grade Algebra II: Applications and Solutions of Quadratic Equations

This was an Algebra II lesson on applications of quadratic equations and use of calculators to solve quadratic equations. In the previous lessons of this unit, students had learned to solve quadratic equations by factoring and by completing the square. The teacher indicated that while the course content and sequence is defined by the district and state curriculum standards and the designated textbook, his instructional decisions were shaped by his perception of the students' low ability (several students were repeating the course), by his opinion that the book is too hard for them, by his desire to see them succeed on tests, and by time constraints.

The entire lesson was taught in a whole-class, lecture format. The teacher selected four textbook problems and modeled solutions for the students. The problems were:

1. By how much do you need to extend the dimensions of a 10×6 rectangle in order to double its area?
2. How can you double the area of a 4×4 square?
3. n is greater than its reciprocal by 1. Find n.
4. A rectangular field has an area of 5000 m^2 and is surrounded by a 300-m fence. Find the dimensions.

For the first problem, the teacher drew a diagram and translated the verbal description of the word problem into a quadratic equation. He demonstrated the calculator keystroke sequence to use in finding the square root of a positive number. The teacher wrote the value for the positive value of the variable without mentioning the negative value. He followed a similar method for the remaining three problems, asking a few questions, but answering most of them himself. Throughout the lesson, students attentively watched the teacher and took notes. As time ran out, the teacher assigned the next two textbook problems for homework.

10^{th}–11^{th} Grade Algebra Tech II: Central Tendency—Mean, Median, and Mode

The lesson in this Algebra Tech II class of 10^{th} and 11^{th} graders came near the end of a unit on statistics. The teacher designed this 90-minute lesson to provide an opportunity for students to finalize their study of statistics terms. She purposefully included basic mathematics skills—division, fractions, decimals, and percents—in the lesson because she is trying to prepare the students for the required state exit exam. There were three main segments in the lesson; first, the teacher led a whole-class review of a worksheet given for homework; second, the teacher gave the students another review worksheet to complete in class; and, third, students were given time to work on their independent statistics projects.

The lesson began with the teacher asking students, “What are the measures of central tendency?” After students identified mean, median, and mode, the teacher asked them to get out their homework worksheet. The homework worksheet contained problems on finding mean/median/mode of data sets, determining median and mode from a line plot, and reading stem and leaf plots, and it ended with a couple of word problems. The teacher called on each student to provide an answer for a homework problem, but approximately half of the student answers were incorrect. If the student she called on had not done the problem, then class discussion came to a halt while the teacher waited for the student to produce an answer. Most students had problems with the stem and leaf plots; they either couldn’t determine median and mode or converted the stem and leaf plot to a list in order to arrive at an answer. The teacher was not concerned about how students got an answer as long as the answer they got was correct. The teacher summed up this part of the lesson by giving a couple of examples illustrating when measures of central tendency might be helpful (e.g., planning which meat to buy for a barbeque based on a survey of 50 people).

The teacher distributed four worksheets. She explained that they were to do the first three worksheets in class and the fourth sheet for homework. Students were directed to work alone on the worksheet pages. In general, students could successfully complete routine calculations of mean, median, and mode but had a great deal of difficulty with matching statistics terms with descriptions (e.g., “A measure of variability used to compare such things as the temperature differences of the warmest and coldest days of the year.”). Students who could not figure out an answer typically guessed rather than ask the teacher for help. By the time the class ended, nearly all the students were working on the required bar graph of their survey data from their independent statistics project, mostly an exercise in drawing and coloring. As the bell rang, the teacher reminded the students several times that they only had to do problem #2 on the homework worksheet.

10^{th}–11^{th} Grade Honors Pre-Calculus: Trigonometric Functions

The Honors pre-calculus lesson for sophomores and juniors focused on trigonometry. Previously, students had studied definitions of trigonometric functions, the unit circle, and right angle trigonometry. In this lesson students learned how changing parameters in the equation $y = d + a \sin b (x - c)$ affects the shapes of the corresponding graphs. The lesson was taught because it fit into the logical sequence given in the department-adopted textbook and the teacher considered it to be important content for the majority of students in this class, students who would go on to Calculus and Physics classes the next year.

The lesson began with review problems to reinforce the definitions of the six trigonometric functions and their relationships (e.g., Find the quadrant and the values for the other five trigonometric functions of an angle if its cotangent is $-\sqrt{3}/3$ and its cosine is greater than 0). The teacher wrote on the board "Amplitude, Period, Phase Changes of Sine and Cosine." She drew the unit circle and labeled some of the special angles. She used a Slinky to model the wave nature of sine and cosine functions and discussed independent and dependent variables, the period of the function, and its amplitude. The teacher led a brief discussion of the Doppler effect as an application of trigonometric functions. She demonstrated how to graph $y = \cos x$ using its maximum, minimum, and intercepts and asked students to describe differences between the graphs of the sine function, $y = \sin x$, and the cosine function, $y = \cos x$; students noticed that they are the same shape but with an offset. The teacher led the students through a series of one-parameter changes (e.g., $y = 3 \sin x$, $y = \sin 2x$, $y = 2 + \sin x$, $y = \sin (x + \pi/2)$. The class discussed the effects of a, b, c, and d in the equation $y = d + a \sin b (x - c)$. Together the class created the function $y = -3 + 4 \sin 6(x + \pi/3)$, and the students computed the amplitude, period, and horizontal and vertical shifts of the graph of this equation. The class ended with a quiz on the unit circle.

Except for the time when students were working independently on the quiz, they were engaged in a whole-class discussion of review and new material. The teacher typically called on a student, the student answered, the teacher told the student whether the answer was right or wrong, gave the student a chance to correct the answer if necessary, or called on another student. The teacher encouraged students to participate in the class discussion. Students were attentive throughout the lesson.

10^{th}–12^{th} Grade Algebra II: Binomial Expansion

The teacher taught this lesson on binomial expansion because this content is mandated in the state and district objectives for Algebra II and because she decided that the content fit well in the instructional sequence. As the lesson began, students knew they could expand a binomial like $(2x - 3y)^5$ by using $(2x - 3y)$ as a factor five times or by using Pascal's triangle method. In this lesson the teacher wanted to teach students another way to expand binomials.

The lesson began with a warm-up activity in which students were asked, first, to identify terms, coefficients, and degrees in a four-term polynomial expression and, second, to classify four polynomials as monomial, binomial, trinomial, or other. After allowing a few minutes for students to work independently, the teacher led a whole-class discussion of the warm-up problems. As an additional review, students participated in a whole-class game of Jeopardy in which the teacher would hold up a card, and a student would call out the appropriate question (e.g., for the card $a^m a^n$, the student asked, "What is a to the (m plus n)?"). Using an interactive-discussion format, the teacher reviewed special products of binomials (e.g., $(a + b)^2 = a^2 + 2ab + b^2$) and how the distributive property is used in multiplying polynomials.

The teacher put a copy of Pascal's triangle on the overhead and reviewed Pascal's triangle procedure for binomial expansion using the example $(x + d)^4$. The teacher carefully stated each step in the procedure—"write the coefficients, write the exponents, put the signs between the terms." The teacher continued the review by working two more problems of this type. The teacher told the students that she wanted them to watch her closely as she expanded the binomial, $(x - 2y)^5$, by a different method, the binomial theorem method. She stressed that they were to watch her, without taking any notes, and she promised to give them a handout containing notes on the method after she finished demonstrating the method. The teacher demonstrated a well-defined procedure without providing any explanation of why the procedure worked. As a second example, she selected a problem that she had done the previous day using Pascal's triangle method and worked it this time using the binomial theorem method. Students agreed that the final answer was the same by the two methods.

The teacher distributed notes on the binomial expansion method and a worksheet containing ten binomial expansion problems. Students were asked to work in groups to complete the ten problems, by either method, and to turn in one set of solutions per group at the end of the class period. The teacher provided some help to students as they were doing group work, but mostly, she was keeping a check on the groups' progress toward completing the assignment. Students in each group tended to work on different problems so they could finish the assignment more quickly. Some students watched as other students did the assignment. The teacher interrupted the group work to demonstrate to the class how to do one of the problems using the Pascal's triangle method. The lesson ended with students turning in the group assignment, whether complete or incomplete.

11th–12th Grade Pre-Calculus: Finding Roots of Polynomial Equations

The topic of this lesson, finding roots of polynomial equations and relating roots to graphs, was chosen because it was part of the Pre-calculus curriculum and followed sequentially from the previous lesson; it was also the next lesson in the textbook. The teacher had personally chosen the textbook based on his perception that it was the best match for the district curriculum and as preparation for the calculus course that most of the students would take the next year.

The lesson began with a short warm-up review exercise which required students to divide polynomials using synthetic division and to identify possible graphs for given polynomial equations. After he had answered all the student questions on the review, the teacher introduced a short worksheet on zeros of polynomials. The students were given two tasks: (1) to use the graphing calculator to graph three polynomials, identify the zeros in each polynomial, and write each equation in factored form and (2) to write a polynomial equation that could be an equation for the graph shown in the fourth worksheet problem. During this exercise the teacher circulated among the students to monitor their work. At one point, he brought the group back into a large-group discussion to review the use of synthetic division.

During the next segment of the lesson, students were given a set of graphs and asked to predict the nature of an equation that would result in a graph with that set of characteristics. Again, the teacher moved among the students to monitor their solutions and to create small discussion groups to look at each other's work and to discuss solutions. When students had completed their work, the teacher led a large group discussion. Students shared their predictions and discussed their work. The teacher probed student responses and involved other students in suggesting alternative solutions and rationales for these solutions. The teacher introduced two polynomials and asked the students to solve one using the graphing calculator and the second using a non-calculator procedure. Again, the teacher monitored student work and facilitated a whole-group discussion of the process and mathematical reasoning underlying the non-calculator method. The teacher assigned some homework problems from the textbook, and the students worked on this assignment, individually or in small groups, until the end of the period.

11th–12th Grade Statistics: Working Probability/Combination Problems

The lesson focused on problem solving in the area of probability, specifically combinations. The course was a teacher-designed statistics course for high school juniors and seniors that was offered because the state's high school exit exam contained statistics questions. This course, offered as an elective in mathematics, was put into the curriculum in response to a sense that there was not enough time in existing mathematics courses to adequately address statistics concepts. Definition of the curriculum for this course was totally the responsibility of the teacher; the teacher's development of the curriculum was based on finding common themes and problems in "seven or eight" introductory statistics books. The students had not been issued a textbook.

During the previous lesson the teacher had given students a mini-lesson on procedures to use in working probability/combination problems, and he had given them four worksheet pages of practice problems drawn from two statistics textbooks. As students entered the classroom they moved desks around to form clusters as needed for their own group while the teacher continued grading papers at his desk. The size of the groups ranged from 2 to 6 students. Students each got a graphing calculator from the teacher's desk and picked up their class folder from the side of the room. Students got out their copies of the worksheet pages and began working, within their groups, on the problems. The teacher announced to the class that he was preparing a homework assignment for them on probability and that the homework assignment would be due in two weeks. Other than this announcement the teacher was not involved in the lesson except when asked questions by students. When a student asked a question about a particular problem, the teacher, without looking at the student's work or asking the student what he had tried, would work the problem on the board and then check the answer key to see if he got the correct answer. If other students noticed the teacher working a problem on the board, they would copy his solution. Most of the students tried to apply the procedures the teacher had shown them the previous day. Some students were not working on the worksheets; the teacher did not attempt to engage these students in the task. The students spent the whole period working on the set of worksheet pages; the teacher did not debrief the lesson or provide any closing comments.

12^{th} Grade Advanced Placement Calculus: Differential Equations

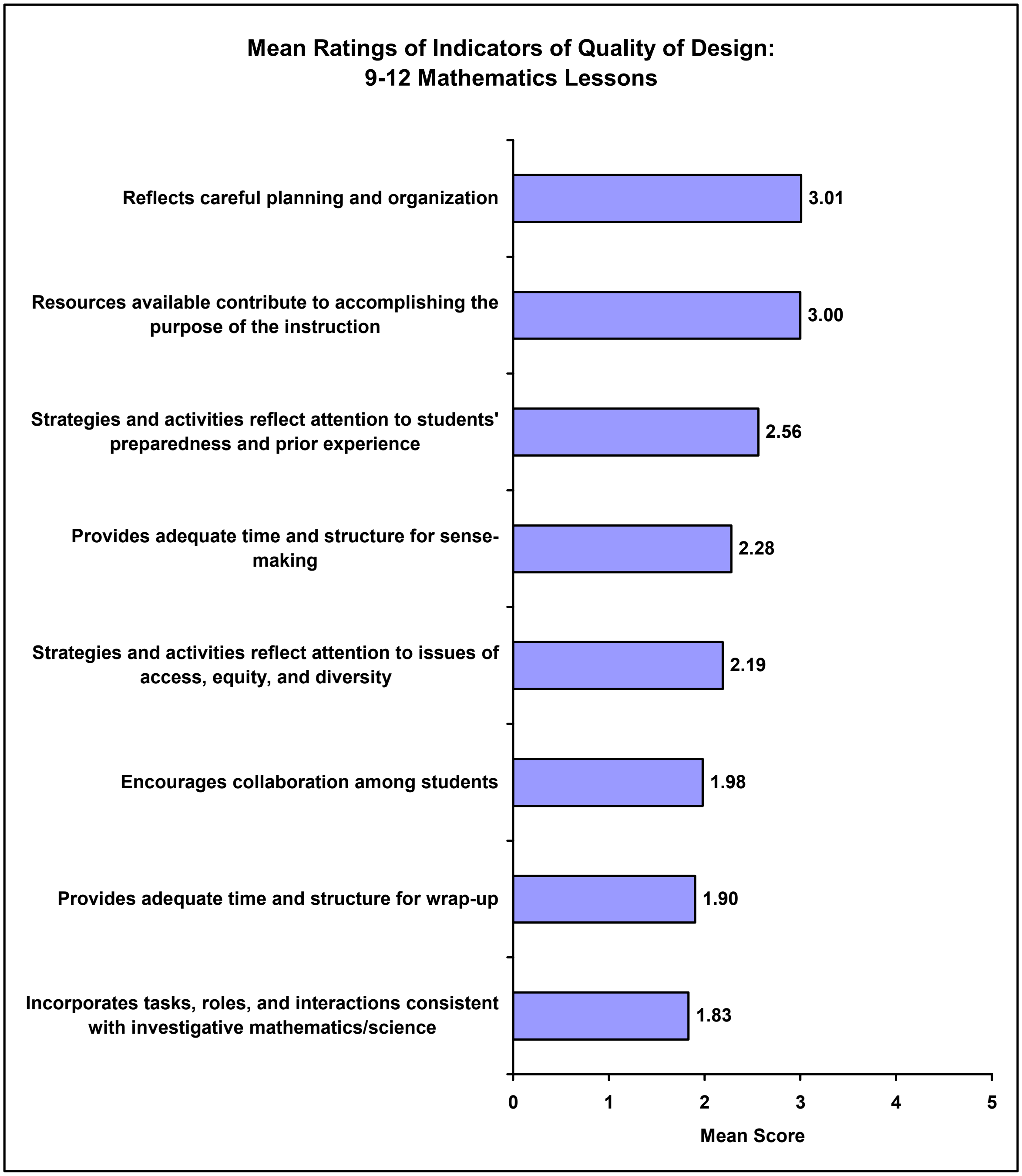
The topics for the lesson—integration, partial separable differential equations, and growth and decay—were selected because they are in the College Board's Advanced Placement Calculus curriculum. The teacher chose the textbook specifically for the problem sets it contains, which he likes because they are similar to the kinds of questions he expects the AP exam to contain, and because they include a number of application problems, such as those emphasized in this lesson. His pedagogy in this lesson included lecture, working examples, guided practice, and independent practice—pedagogy which, in the teacher's opinion, is typical of college-level mathematics instruction. The students in this class were very able, and highly motivated to learn the content of the course.

The lesson began with a set of seven warm-up exercises selected to review recently-taught methods of taking indefinite integrals. Students worked these problems independently or in informal groups or pairs. After giving students the answers to these exercises, the teacher answered students' questions and worked several of the problems at the board. Additional review followed as the teacher gave students the answers to the previous night's homework. Students asked the teacher to work a few of the homework exercises, which he did.

The teacher began a lecture on integrating partial separable differential equations with a few examples (e.g., $dy/dx = (x + 3x^2)/y^2$). He moved quickly to a derivation of the general exponential growth and decay formula, $y = Ce^{kt}$. The remainder of the lecture focused on applications of the growth and decay formula (e.g., "At noon a bacteria population is 10,000 organisms. Two hours later, 40,000 organisms are counted. Assuming exponential growth, how many bacteria are there at 5:00 p.m.?"). As more application problems were presented, the teacher moved toward guided practice for the students, with decreasing guidance for each problem. The teacher made an assignment from the textbook which the students began in class and were to finish for homework. The students worked individually, but often compared answers and frequently asked one another for help with procedures.

Ratings of Lesson Components

The designs of high school mathematics lessons are, on average, most highly rated for reflecting careful planning and organization and for utilizing the available resources to accomplish the purpose of the lesson. High school mathematics lessons are weaker in many areas, including incorporating strategies and activities that reflect attention to students' preparedness and prior experience, and providing students with the time and structure needed for sense-making. Lessons are weakest in encouraging collaboration among students, providing time and structure for wrap-up, and incorporating tasks consistent with investigative mathematics. The relatively low ratings in these areas may explain the low synthesis ratings for lesson design as 69 percent receive low ratings and only 9 percent receive high ratings.



Mean Ratings of Indicators of Quality of Design:
9-12 Mathematics Lessons
Reflects careful planning and organization
3.01
Resources available contribute to accomplishing the purpose of the instruction
3.00
Strategies and activities reflect attention to students' preparedness and prior experience
2.56
Provides adequate time and structure for sense-making
2.28
Strategies and activities reflect attention to issues of access, equity, and diversity
2.19
Encourages collaboration among students
1.98
Provides adequate time and structure for wrap-up
1.90
Incorporates tasks, roles, and interactions consistent with investigative mathematics/science
1.83
0
1
2
3
4
5
Mean Score

Figure E-1

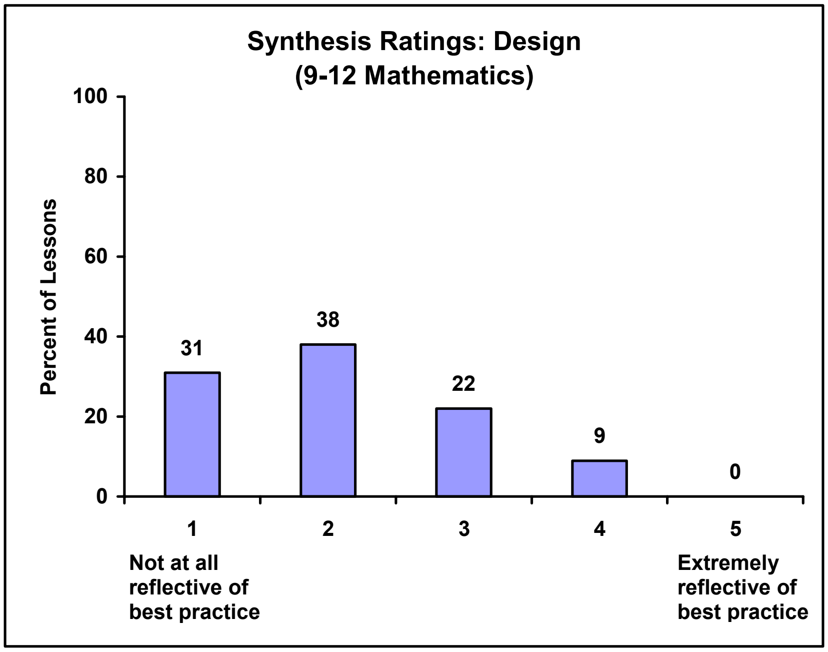


Figure E-2

The implementation of high school mathematics lessons is rated most highly for teachers' confidence in their ability to teach mathematics. The implementation of lessons is weaker in regard to teachers' classroom management and pacing (moving either too quickly or too slowly). High school mathematics lessons are weakest in adjusting instruction according to the level of student understanding, using instructional strategies consistent with investigative mathematics, and posing questions that enhance student understanding. These low ratings are reflected in the implementation synthesis ratings. Seventy-one percent of lessons receive a low rating for implementation while only 12 percent receive a high rating.

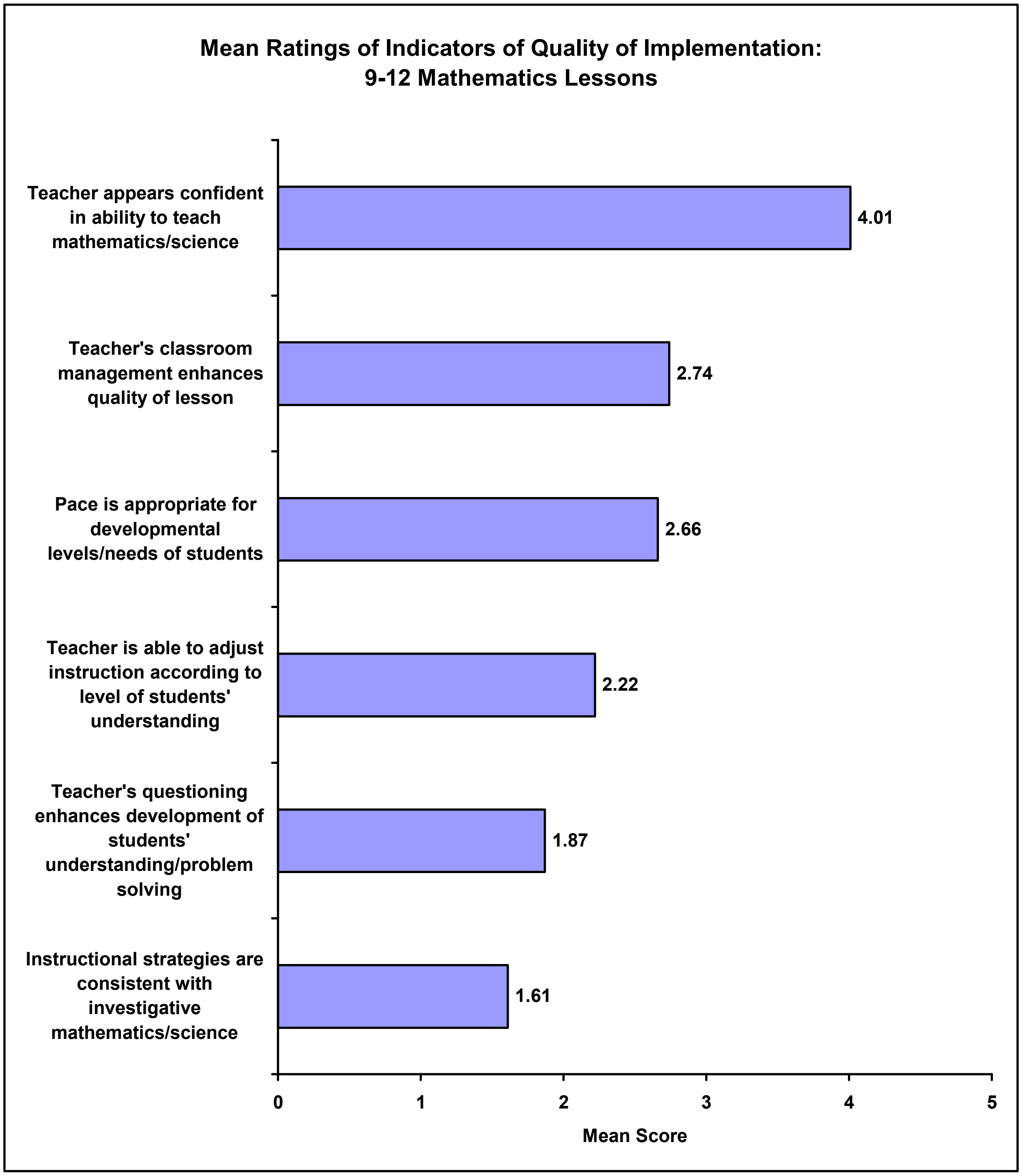

Mean Ratings of Indicators of Quality of Implementation:
9-12 Mathematics Lessons
Teacher appears confident in ability to teach mathematics/science
4.01
Teacher's classroom management enhances quality of lesson
2.74
Pace is appropriate for developmental levels/needs of students
2.66
Teacher is able to adjust instruction according to level of students' understanding
2.22
Teacher's questioning enhances development of students' understanding/problem solving
1.87
Instructional strategies are consistent with investigative mathematics/science
1.61
0
1
2
3
4
5
Mean Score

Figure E-3

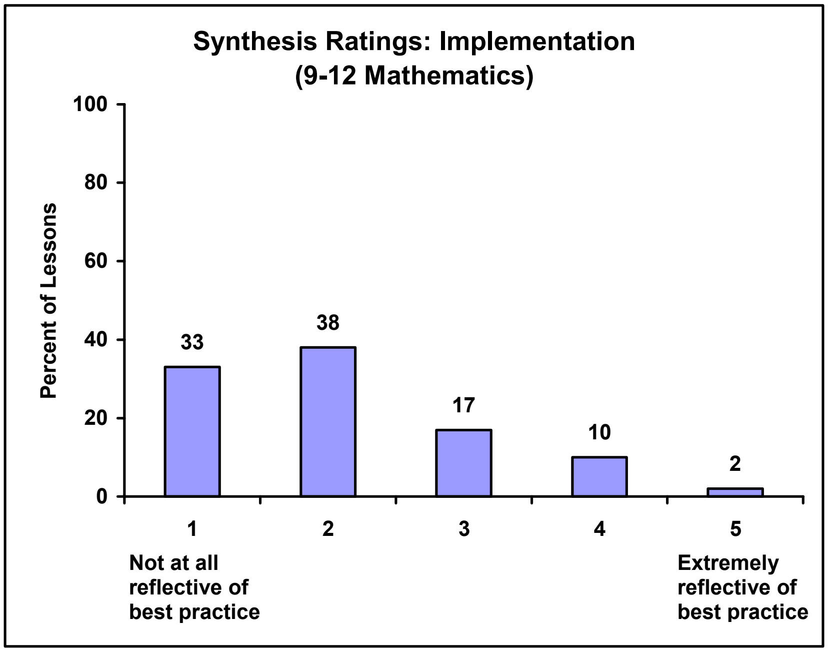


Figure E-4

The content of high school mathematics lessons is, on average, rated highest for focusing on significant and worthwhile content at a developmentally appropriate level and doing so accurately. In addition, lessons are rated highly for teachers displaying an understanding of the concepts. High school mathematics lessons are weakest in intellectually engaging students with important ideas, making connections to other areas, providing opportunities for students to make sense of the content, and portraying mathematics as a dynamic body of knowledge. Seventeen percent of lessons receive a high synthesis rating for content, 30 percent receive a medium rating, and 53 percent receive a low rating.

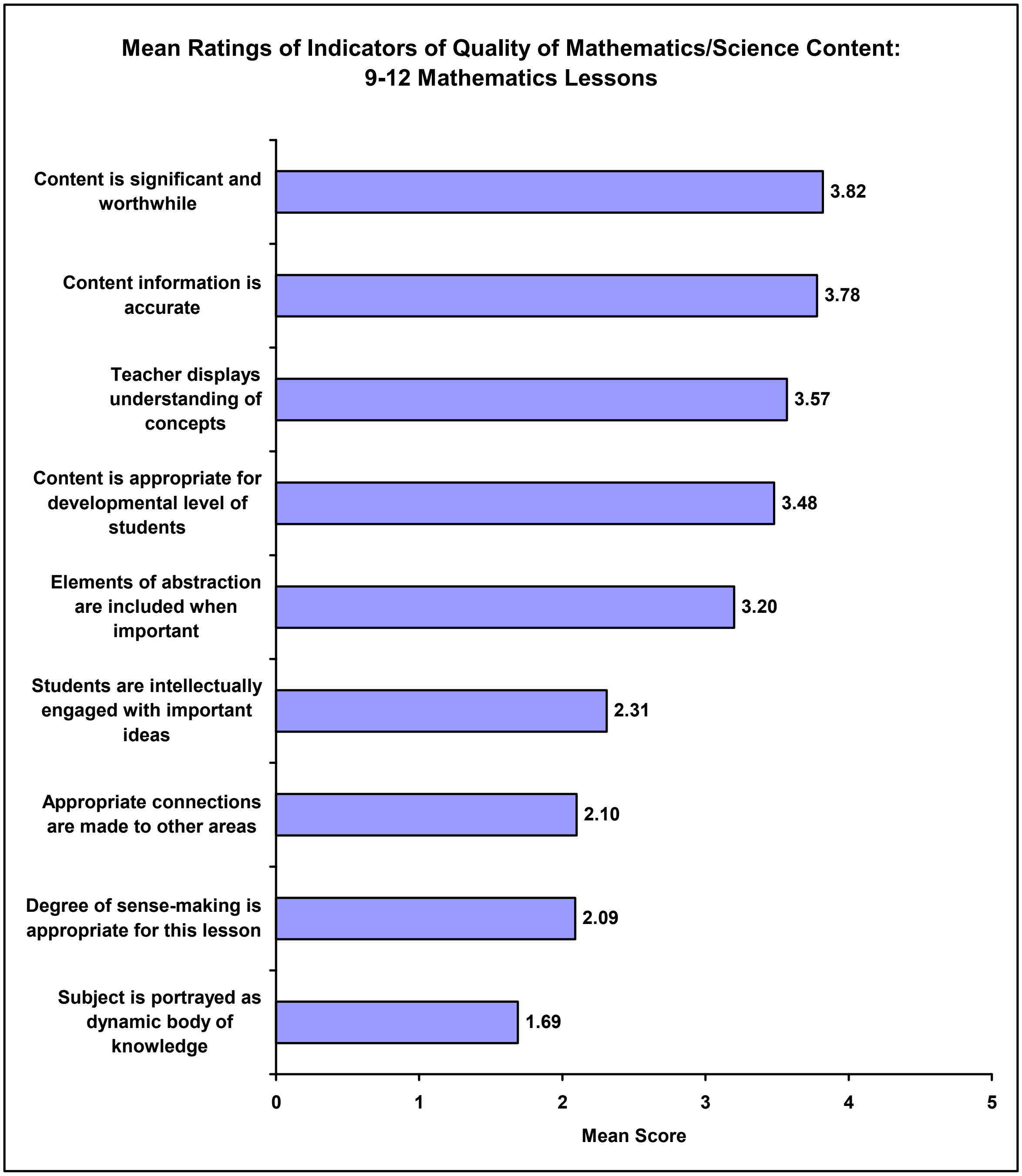

Mean Ratings of Indicators of Quality of Mathematics/Science Content:
9-12 Mathematics Lessons
Content is significant and worthwhile
3.82
Content information is accurate
3.78
Teacher displays understanding of concepts
3.57
Content is appropriate for developmental level of students
3.48
Elements of abstraction are included when important
3.20
Students are intellectually engaged with important ideas
2.31
Appropriate connections are made to other areas
2.10
Degree of sense-making is appropriate for this lesson
2.09
Subject is portrayed as dynamic body of knowledge
1.69
0
1
2
3
4
5
Mean Score

Figure E-5

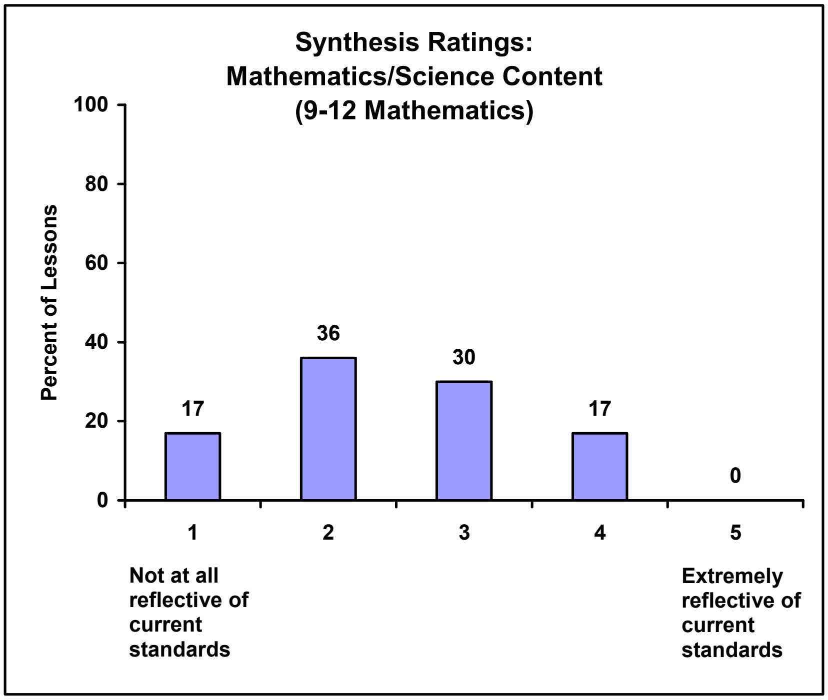


Figure E-6

In regard to classroom culture, high school mathematics lessons are rated strongest in having a respectful climate. Lessons are weakest in encouraging students to generate ideas and questions and in their level of intellectual rigor. The synthesis ratings for classroom culture reflects these indicators with 13 percent of lessons receiving a high rating, 22 percent receiving a medium rating, and 65 percent receiving a low rating.

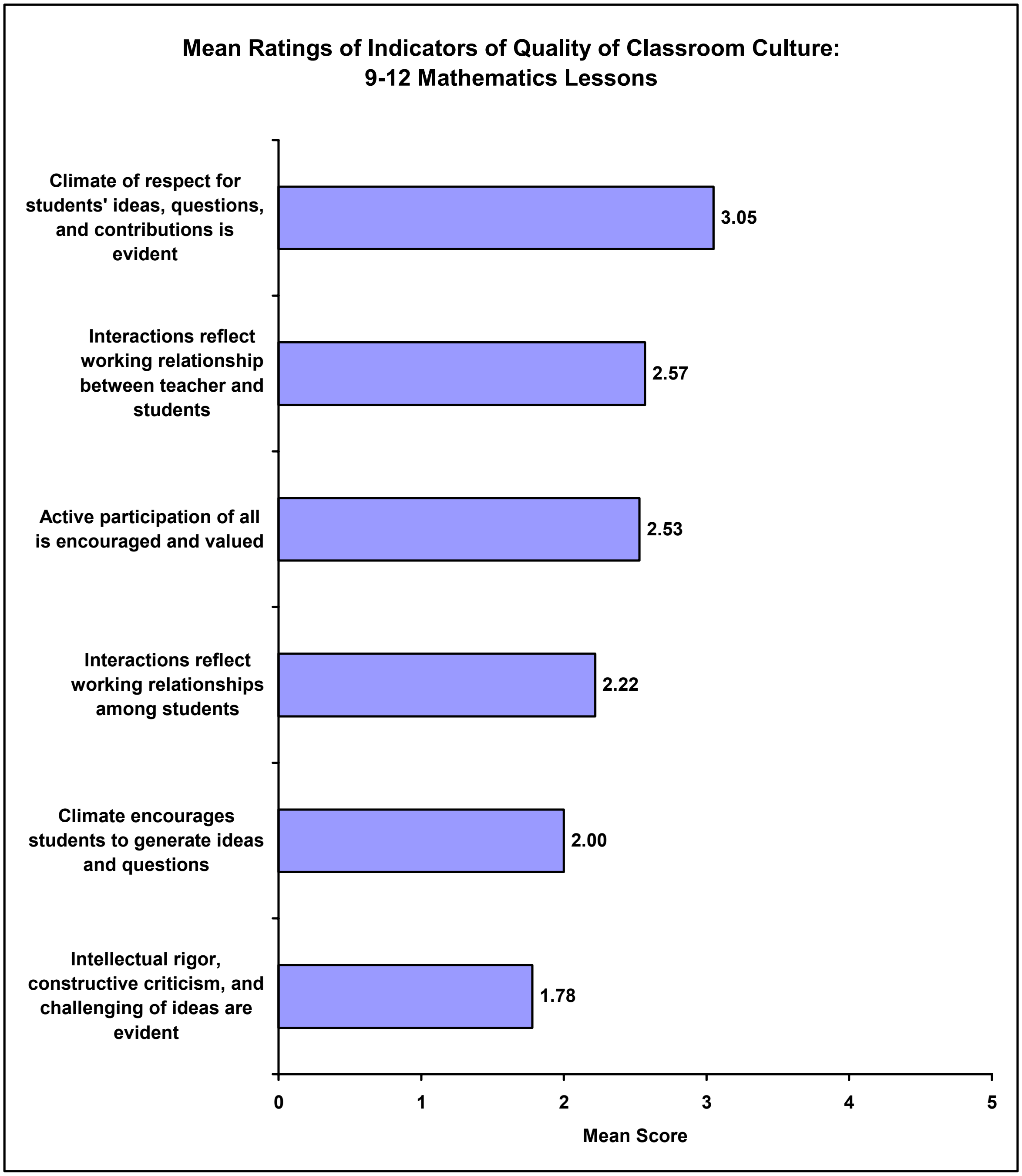

Mean Ratings of Indicators of Quality of Classroom Culture:
9-12 Mathematics Lessons
Climate of respect for students' ideas, questions, and contributions is evident
3.05
Interactions reflect working relationship between teacher and students
2.57
Active participation of all is encouraged and valued
2.53
Interactions reflect working relationships among students
2.22
Climate encourages students to generate ideas and questions
2.00
Intellectual rigor, constructive criticism, and challenging of ideas are evident
1.78
0
1
2
3
4
5
Mean Score

Figure E-7

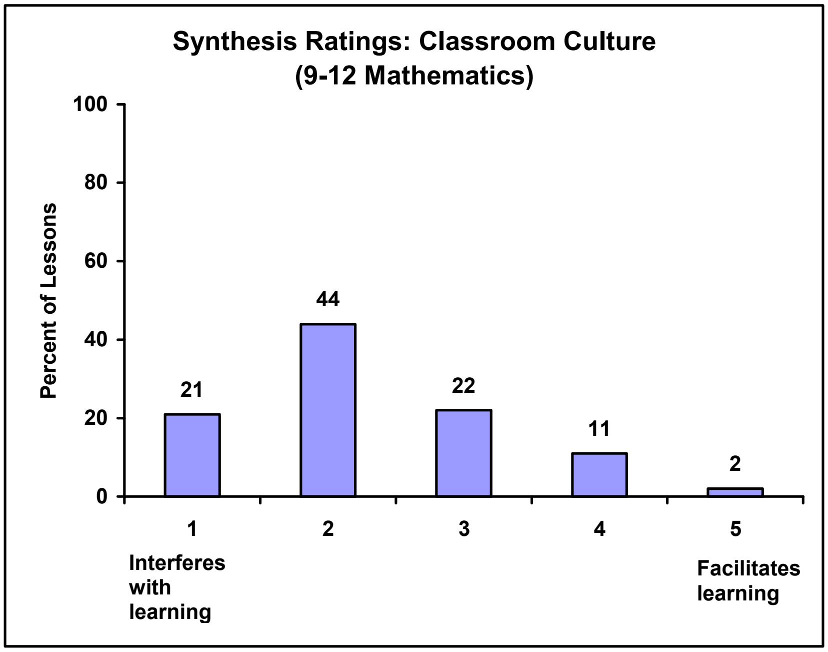

Synthesis Ratings: Classroom Culture
(9-12 Mathematics)
100
80
60
40
20
0
Percent of Lessons
21
44
22
11
2
1
2
3
4
5
Interferes with learning
Facilitates learning

Figure E-8

Overall Lesson Quality

Following the ratings of the individual components of the lesson, the researcher was asked to consider the likely impacts of the lesson as a whole. Less than a third of the lessons have positive impacts on students' understanding of mathematics or confidence to do mathematics. Sixteen percent or fewer lessons have a positive impact on students' interest in the mathematics, ability to apply the skills and concepts they are learning to other disciplines or real-life situations, and capacity to conduct mathematical inquiry. The majority of lessons are likely to have a negative impact on students' understanding of mathematics as a dynamic body of knowledge generated and enriched by investigation, a reflection of the rote, algorithmic methods being used in most mathematics lessons. (See Table E-1.)

Table E-1
Likely Impact of the Lesson: Mathematics 9–12

	Percent of Lessons		
	Negative Effect	**Mixed or Neutral Effect**	**Positive Effect**
Students' understanding of important mathematics/science concepts	19	51	30
Students' self-confidence in doing mathematics/science	24	48	27
Students' interest in and/or appreciation for the discipline	44	41	16
Students' ability to apply or generalize skills and concepts to other areas of mathematics/science, other disciplines, and/or real-life situations	29	59	12
Students' capacity to carry out their own inquiries	28	61	11
Students' understanding of mathematics/science as a dynamic body of knowledge generated and enriched by investigation	61	30	8

Figure E-9 shows the percentage of 9th–12th grade mathematics lessons in the nation rated at each of a number of levels. (See page 9 of the Observation and Analytic Protocol in Appendix A for a description of these levels.) Sixty-nine percent of high school mathematics lessons are rated as low in quality on the capsule rating, 23 percent are rated as medium in quality, and 8 percent are rated as high in quality.

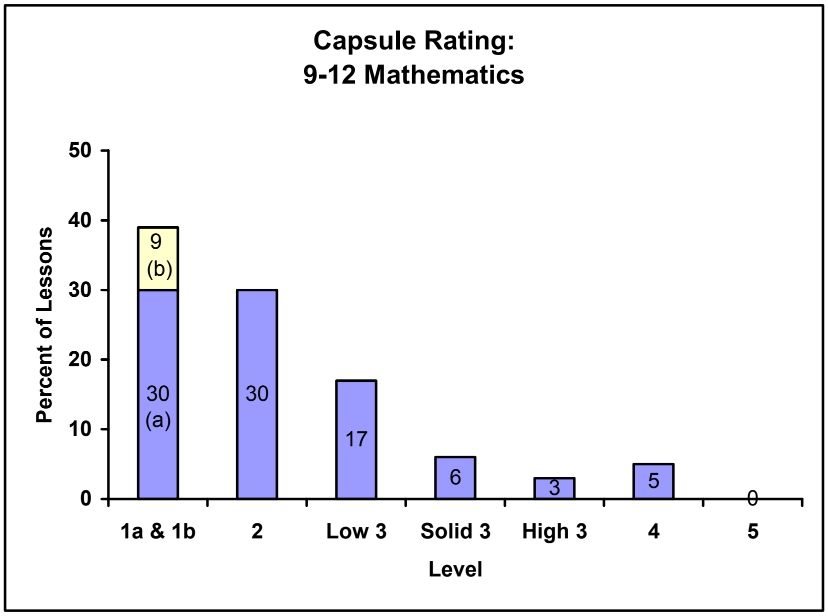


Figure E-9

The following illustrate lesson descriptions that were rated low, medium, and high in quality.

Sample Low Quality Lesson: Passive "Learning"

This lesson was the beginning of a unit on equations and inequalities in a 9th grade class. The lesson was designed to provide students with an introduction to solving equations.

After roughly ten minutes of taking attendance and returning graded assignments to the students, the teacher introduced solving linear equations by working through several examples on the overhead projector. The teacher appeared to know the content and used correct terminology. However some examples, such as $\square + 5 = 17$, appeared to be too simple for these 9th grade students.

In the teacher's effort to help students better understand how to solve the equations, he merely asked them to remember and repeat the procedures he demonstrated. The teacher's presentation of the content included questions and comments such as, "There's the variable, what's the opposite?" and "Tell me the steps to do." He did very little to engage students with the content; two students slept through the teacher's entire presentation, and one read a magazine. Other students contributed very little but instead spent time asking about the particulars of the upcoming assignment.

Upon completing his presentation of the content, the teacher handed out a worksheet for students to complete individually. He then read the answers for students to check their work. With no further discussion of potential problems or any indication of how well students understood the mathematics from the first worksheet, the teacher handed out three more worksheets for students to complete during the remaining 65 minutes of this block lesson. While the students were completing the worksheets, the teacher monitored what students were doing and occasionally assisted individual students. Students who finished before the class ended talked for the last 15 to 20 minutes.

Overall this lesson was very poor in helping these 9th grade students better understand the mathematics involved in solving linear equations. The teacher's presentation of the content was procedurally-focused and emphasized memorizing and duplicating the steps he defined for solving the problems. The examples he provided failed to challenge many of the students, and his questioning and management style did very little to engage the class. Further, the excessive amount of time dedicated to individual practice, the lack of opportunity for students to discuss their work or interact with one another, and the absence of any type of wrap-up of ideas seriously hindered students' ability to make sense of the content.

Sample Low Quality Lesson: "Activity for Activity's Sake"

Students in this high school Geometry class had nearly completed a unit on congruent triangles. The bulk of the class time was spent in review, copying and constructing geometric figures using only a compass and a straight edge. One new construction was added, that of constructing a perpendicular line from only a point and a line segment.

The teacher began the lesson by telling students to get out their index cards, references that the students had created to help them remember proofs and the procedures for various geometric constructions introduced thus far this year. She then passed out a worksheet containing five construction problems. She instructed the students to do the first problem (constructing or copying a line segment) and then suggested that they look at the instructions on their index cards.

After a few minutes, the students were told to go on to problem number two. There had been no large group review of the first problem, but the teacher had walked around the room looking over their work. At one point she said, "I want to see all your markings. Label them."

Student desks were grouped together in clusters of 4–5 to make "tables." Some students talked to their neighbors during the seat work; others did not. Students had their bundles of index cards out on their desks and appeared to be using them. They were required to complete the worksheets individually, though the teacher encouraged them to ask for help from their peers. ("Look around…if you're having trouble, if you can find someone who knows, you can ask them.")

Students began to talk more and the teacher, noticing some behavior she wanted to correct told them, "Do not move on to number three. Do *not* make helicopters out of your protractor." After about 25 minutes, the activity changed. The teacher instructed them all to get out their textbooks and copy onto one of their index cards the three-step process for constructing a perpendicular line given only a point and a line segment.

After students had finished copying the instructions, the teacher asked, "How many of you think you could do it following the instructions? Try it. Use my worksheet." She told them to be careful not to change the compass (the length of the arc), but there was no discussion about why this would be important in the construction.

This lesson did not provide much of an opportunity for students to deepen their understanding of mathematics. Construction was discussed in strictly procedural terms and the concept of congruence was completely missing. The only mention of keeping a measurement consistent in these drawings completely missed the mark in terms of furthering students' understanding of the role of congruent triangles in verifying constructions made without a protractor or ruler. There was no evidence of higher-order thinking.

Sample Medium Quality Lesson: Beginning Stages of Effective Instruction

This 8th grade class had been studying systems of equations and most recently had completed a test on the topic. Unhappy with the results, the teacher chose to return to the topic for this lesson.

The lesson was designed around the school's adopted text, although the teacher also incorporated examples from another textbook that he had found in his room at the start of the year. The lesson began with the teacher writing the definition of systems of equations on the board and beginning to review three methods for solving systems of equations: substitution, addition/subtraction, and multiplication. The teacher talked through the different methods and worked one example. The review was spirited and well-organized, with ample student participation. The teacher's questions were appropriate for the task and kept students thinking; the students raised questions and from time to time asked the teacher to slow down. Overall the review activities provided guidance that seemed to benefit students as they worked individually.

After the review, the teacher gave the students two systems to practice individually:

(Add/Subtract)	(Multiply)
$y + x = 3$	$2x - y = 6$
$y - x = 1$	$-3x + y = 1$

Each system was titled by the specific strategy that was to be used to solve it. Although this appeared to be a bit too much guidance for some students, a number of students may have needed this level of directedness initially. Students were attentive and focused during individual practice time, and the time allotted for students to complete the problems was sufficient. The teacher left students to grapple with the content and was hardly ever called for assistance. He followed the individual work with students going to the chalkboard to complete and explain the different problems. Students eagerly volunteered to come forward and talked through their work. During the students' presentations the class listened attentively and often asked questions. The teacher probed for student understanding and emphasized clarity of language.

Most of the teacher's questions were focused on the procedural steps for solving the system, for example, "When I add a –3x to a 2x, what do I get?" and "I multiplied every term by…?"; However some questions were a little more thought-provoking and less leading, for example, "Since I'm looking for +y, how am I going to change –y = –4?"

After going over the first problems that students did individually, the teacher wrote three more problems on the board from the supplementary textbook. The problems were similar to the ones listed above, and again students were asked to complete the three problems individually. The discussion went just the same as the prior one.

This pattern—the teacher giving two or three problems then having the students work them on the board and explain—continued until the end of the period. Although sense-making was tended to throughout the lesson, there was no wrap-up of the topic. Overall, the lesson was built on strong content but had a flawed design that involved the re-teaching of a topic in the exact same way it was taught the first time. The students were not given the opportunity to connect the topic to other subjects, nor to see where the topic fit in the subject of algebra.

Sample High Quality Lesson: Traditional Instruction

In the lesson prior to this one, this Geometry class had been introduced to two theorems: the Dual Perpendicular Theorem and the Dual Parallel Theorem. The class had also worked on defining parallel lines. This lesson was designed to review these ideas because many students were absent due to a school field trip. An additional purpose of the lesson was for students to better understand the fundamentals of compass and straight edge constructions.

The teacher started the lesson with an activity designed to help students review the skills that had been taught during the prior lesson. Students were asked to copy both theorems that the teacher had already written on the board and to list five ways of showing that two lines are parallel to one another. After students completed copying the theorems and making their list, the teacher began to ask questions of the students to check their understanding. Of particular notice was the depth of the teacher's questions and his wait time. For example, the teacher asked students why the word "coplanar" was included in one theorem but not the other. He probed further as to whether the word was necessary for one or both theorems and had students provide examples that helped support their thinking. Other questions during this segment ("Why are *l* and *n* parallel?"; "How do I illustrate that?"; and "Can you think of another way of showing that?") were similarly rich in getting students to think more deeply about the mathematics content as well as helping them connect the concepts to previously-learned material.

Following this discussion, the teacher had all students go to the dry erase boards that surrounded the room to simultaneously practice problems. The problems called for students to recall geometric concepts that had been previously covered in class, as well as to discover the ways these concepts connected with what they were presently learning about the theorems. Students appeared to be very comfortable listening to the teacher's set of directions for each problem, and for the most part students were able to solve the problems and identify connections. Students worked individually, but looked to their neighbors on either side for guidance and approval. The teacher also served as a guide and checked the work of each student. As the activity progressed, he paid closer attention to those students who appeared to be having more difficulty completing the problems.

After walking the students through several problems, the teacher began the new unit on basic constructions. Students took out their notebooks and were given an unmarked straight edge and a compass. The teacher directed them step-by-step through three constructions. As the teacher completed and explained a step on the board, students duplicated the step in their notebooks. The teacher's questioning and pace appeared to keep the content engaging for the class.

The teacher then gave students the option of working individually or in pairs to complete an assignment that required them to apply what they had just learned in creating three additional constructions. During this time, the teacher walked around and provided assistance to those who were struggling.

The design of the lesson appealed to a variety of learning styles and appeared to match the needs of these geometry students. The lesson was centered on content that was rich and appropriate. Throughout the lesson students grappled with the content and continually moved forward in their understanding as a result of the teacher's management style.

Sample High Quality Lesson: Reform-Oriented Instruction

The purpose of this high school pre-calculus lesson was to help students learn to factor polynomials, using both a calculator and a non-calculator method. The lesson began with a short warm-up exercise that was designed to be a review. The activity required students to divide polynomials by using synthetic division and to identify possible graphs for the given polynomial equations. After the teacher answered student questions about the activity, he asked students to use graphing calculators to graph three polynomials, identify the zeros in the polynomials, and write the equations in factored form. Students were also asked to write a polynomial equation that could be an equation for another graph he provided.

During the exercise, the teacher effectively helped students make sense of the mathematics by circulating and assisting where needed. He constantly assessed student understanding and modified his instruction to address issues that emerged as barriers to students' progress. For example, after noticing the trouble many students were having using synthetic division, the teacher brought the group back together for additional review. When one student stated that she could follow the process but did not understand a particular operation, the teacher asked another student to explain the operation. Continuing to make good use of the collective problem solving skills of the group, the teacher solicited another explanation from a third student.

The next segment of the lesson involved students analyzing a set of graphs to predict the nature of the equation that would result in a graph with that set of characteristics. At times the teacher asked small groups to look at each other's work and to discuss their solutions. When the students had completed their work, the teacher led a large group discussion, asking students to share their predictions. As students offered their thoughts, the teacher again probed their responses and involved other students in suggesting alternative solutions and providing their rationales for these solutions.

After going through an additional activity that involved students solving two polynomials, one using the graphing calculator and one without, the teacher assigned homework problems from the textbook. As students started on the assignment, some worked individually while others worked in small groups. Several students talked together about the operations of the calculators, and one student showed the group another way he had found to use his calculator to factor the polynomial.

The design of this pre-calculus lesson effectively kept the interest and engagement of the students. It included content that was appropriate and challenging, as well as tools and instructional approaches that enabled students to focus on the underlying mathematics concepts. Throughout the lesson, the teacher challenged students to think about different strategies by involving them in critiquing each other's work in a non-threatening manner. The teacher posed questions that pushed the students to extend their thinking and also incorporated ample time for sense-making through both large and small group discussions. Overall the lesson was implemented with great skill, and was highly likely to increase student understanding of factoring polynomials.