# Appendix D

## Grades 6–8 Mathematics

**Typical Lessons**

**Ratings of Lesson Components**

**Overall Lesson Quality**

# Typical Lessons

The following lesson descriptions are based on a random sample of 6th–8th grade mathematics lessons.

## 6th Grade: Division with Decimal Divisors

This 6th grade lesson on division with decimal divisors was situated according to textbook design; in the teacher's words, "I'm just following the book, basically." The lesson also included a review of writing fractions as repeating decimals and a "Problem of the Day" that involved students making predictions based on a set of related multiplication sentences.

As the teacher returned graded papers, students got out their completed homework assignment – a set of textbook exercises that involved re-writing fractions as repeating decimals. Students checked their homework as the teacher read aloud the correct answers and answered students' questions about the homework exercises. The students were given a worksheet containing similar exercises to complete individually; the teacher graded the completed worksheets as they were submitted. Students were asked to work individually on the Problem of the Day (i.e., Find the products $15873 \times 7$, $15873 \times 14$, and $15873 \times 21$ and use those products to predict the product $15873 \times 28$). The teacher led a whole-class discussion of this problem.

The teacher introduced the new content material—division with decimal divisors—by modeling an example on the overhead projector. He demonstrated how to move the decimal points in the divisor and dividend the same number of places in the same direction. When asked by a student, "Why do we move it over?" the teacher replied, "The divisor must be a whole number." For the second example modeled on the overhead, students were asked to help by stating each step, one-by-one, in the procedure. A few students asked questions about the procedure (e.g., "What do we do about remainders?"). The teacher assigned twelve similar textbook exercises for homework. Students worked individually on the homework. Students who finished the homework before the end of the period were allowed to work on their assigned art projects.

## 6th Grade: Fractions, Ratios, and Rates

The teacher focused on the topics of fractions, ratios, and rates during this lesson because these topics would be included in the upcoming state assessment tests. The teacher expressed concerns about the ramifications, on both his school's and his own evaluations, if his students did not perform well on the test. Specifically, this lesson was designed to help students tie in their previous knowledge of fractions with the concepts of ratios and rates.

The lesson began with a review of several fraction concepts: definition of a fraction, proper and improper fractions, and equivalent fractions. The teacher asked a student to "prove" to the class that one-half and two-fourths are equivalent by showing both fractions using an overhead circular fraction kit; this was repeated for $^1/_2$ and $^3/_6$, $^1/_2$ and $^4/_8$, $^1/_3$ and $^2/_6$, and $^1/_3$ and $^4/_{12}$.

The teacher began instruction on ratios and rates by asking a student to define "ratio." The student read the definition from the textbook ("a ratio compares two numbers by division"). The teacher asked students to provide examples of different ways to write a ratio; by looking at the textbook, students were able to suggest "3:4," "3 out of 4," and "3 to 4" as equivalent ways to write the ratio expressed as a fraction, "¾." The teacher directed students to write the definition for ratio in their mathematics notebooks and to complete the textbook's "five-minute check" which provided practice on ways to represent ratios.

The teacher put a definition of rate ("a rate is a ratio that compares quantities with different units") on the overhead and asked a student to read the definition aloud. The teacher wrote the formula "total number of units = rate × time" on the board and discussed a couple of rate examples. He then switched back to ratios and showed how "cross multiplication" could be used to determine if two ratios are equivalent or proportional. After working a few examples, the teacher distributed a textbook worksheet. The exercises on the front side of the worksheet focused on writing ratios in different ways, expressing ratios as fractions, and expressing ratios as rates; on the back of the sheet were exercises that involved solving proportions. The teacher asked the students to work on the worksheet for the last 15 minutes of class and complete it for homework. Some students worked independently on the assignment and others together as the teacher provided additional instruction, as needed.

## 6th Grade: Metric Measuring Units

The primary goal for the lesson was to expand the 6th grade students' understanding of metric units and to help them develop a sense of scale and meaningfulness by relating these units to relevant examples from the real world. The teacher used the district-adopted textbook for the instructional materials, examples, and organization of the lesson. She indicated that, in general, she follows the textbook order of topics in teaching the curriculum.

The teacher began the lesson with a whole-class review of the metric units for length (m, cm, dm, km, mm), capacity (L, kL), and mass (g, kg). Students were asked to draw a two-column chart with the headings "standard" (i.e., U.S. Customary) and "metric." From a collection of grocery store items on her desk, the teacher read information off the labels and asked students to enter that information into their charts (e.g., for the first item the teacher read "1.01 ounces" and "30 milliliters"). The teacher quickly called out several other pairs of measures found on the labels of a variety of products; the students continued to enter the measures into their charts. She asked a couple of students to contribute another pair of measures from the grocery items that they had brought from home.

The teacher asked students to think about the relationships between sizes of containers and measures on the labels. Most of the class seemed to think that bigger packages would contain a larger amount and thus carry a larger number on the label. Comparisons of measures focused on magnitudes, but not on units, so at times students were comparing the mass of one package with the capacity of another package. At one point in the discussion the teacher held up an 8-ounce can of beans (also labeled in grams) and an 8-ounce bottle of Pepsi (also measured in milliliters) and explained to the class that these two items "have the same measure in standard units but different metric units." For the two items there was no discussion of the fact that measures of different quantities were reported on the two packages (i.e., weight and mass were given on the can of beans while capacity was given on the bottle of Pepsi).

The teacher asked the class a series of questions from the textbook (e.g., "How many kg in a gram?" and "Which unit—liters, milliliters, or kilograms—is appropriate for representing the capacity of a milk container? A swimming pool?"). The teacher wrote the answers on the board and students copied the answers down on their papers. About ten minutes before the end of the period the teacher assigned a set of similar exercises (e.g., "What units would you use for bottled juice?") for the students to complete independently. Students worked on these exercises and the homework assignment for the remainder of the period.

## 7th Grade: Number Patterns, Fractions, and Percents

The lesson was a fast-paced review to pull everything together at the conclusion of a 7th grade unit on number patterns, fractions, and percents. The teacher planned this lesson because the students had done poorly on a recent test. The lesson was designed to help students identify their own weaknesses and to provide opportunities for them to deepen their levels of understanding. The teacher was particularly concerned that her students develop a better understanding of the content of the lesson because the topics are a part of the district's prescribed mathematics curriculum and framework, and the topics are on the mandated state-level 8th grade assessment test.

The lesson began with a whole-class, five-minute warm-up exercise—a review of division and fractions—followed by a homework check. The teacher then used the newspaper to discuss current stock market quotes. (The class was participating in a simulated computerized stock market activity in which the students worked in teams of four and bought and sold stocks and competed against other teams around the state.) In the discussion of current stock quotes and trends, the teacher pointed out that stock quotes are being converted to decimals and shortly all stocks would be reported in that way.

The students participated in a whole-class flash card review of the decimal equivalents of common fractions. After the students had checked their answers, the teacher reminded them that they would be taking a placement test the next day to determine their mathematics course next year and that they needed to use the results of this exercise to guide their study for that test.

The teacher instructed the students to complete the textbook worksheet, "Using Number Patterns, Fractions, and Percent," that they had worked on in an earlier class. She organized students in pairs for this part of the lesson, trying to pair students she thought would be able to help one another.

The final activity was a modeling of the concept of positive and negative integers and absolute value. The teacher asked several students to line up along a number line that was drawn on the floor in the center aisle of the room. One student marked zero and others took either positive or negative positions. She involved other students by having them define the "value" of each student, in positive or negative terms, and then explain what the absolute value was. The students shifted positions and the process was repeated once more before the period ended.

## 7th Grade: Percents, Fractions, and Decimals

The lesson on percents, fractions, and decimals was taught by a first-year, un-credentialed teacher. The teacher used the district's 7th grade curriculum and adopted textbook in planning this lesson. She seemed confident in her mathematical content knowledge (she had been an accountant before entering the teaching profession). Her students were used to an established classroom routine that began with practice problems for the district's quarterly assessment tests, followed by going over homework, doing some type of mathematics activity, and completing practice worksheets.

The lesson began with a worksheet that served as a general review of equivalent fractions, the distinction between rational and irrational numbers, and calculating percent increase and decrease. The teacher looked over students' homework (a set of converting-percents-to-fractions exercises) while the students worked on the review worksheet. She called on students individually to provide answers to all but the last two worksheet problems. She decided to omit any discussion of these last questions because they involved calculating percent decrease, a topic that the class had not yet studied. She asked students to exchange homework papers and grade each others' work. If a student raised a question about a homework answer, the teacher solved the problem on the overhead, explaining each step as she wrote.

The teacher had planned a group activity for this lesson but changed it to a whole-class demonstration because of time. Three students were asked to come to the front of the class. One student measured the height and arm spread of a second student, while the third student wrote the numbers on the board. The ratio of arm spread to height was written as a fraction, and the teacher asked the class to calculate the corresponding percent. The teacher demonstrated and discussed the proportion method for calculating the percent (i.e., $59/64 = x/100$). Students were then instructed to complete a textbook practice worksheet that included expressing decimals as percents and percents as decimals and comparing percents and decimals (<, >, or =). The teacher helped students individually as needed but provided no closure to the lesson.

## 7th Grade: Prime and Composite Numbers

The lesson, which followed a lesson on divisibility rules, was designed to introduce students to prime and composite numbers, prime factorization, and factor trees. The teacher's choice of content for this lesson was based on the NCTM standards and the state's mathematics goals for 7th graders. The teacher explained that, as she plans lessons, she goes through the national and state standards to pick out a topic and then chooses related instructional materials from the district-adopted textbook.

The teacher began the lesson with a whole-class review discussion focused on finding areas of rectangles. She reminded students of the formula "A = L × W" and asked the students to find the areas of four rectangles (e.g., "L = 11 in, W = 7 in"). The teacher helped students as they worked on the exercise, after which she asked individual students to put solutions to each problem on the board.

In the second segment of the lesson the teacher asked students to work in cooperative groups on the problem: "Lois, Trisha, Mark, and Dean traded stamps. Lois had 38 stamps when the trading was over. She knew she had given 9 to Trisha and received 11 from Mark and 13 from Dean. How many stamps did she start with? (Hint: Start with 38 stamps and work backwards.)" Each group solved the problem, wrote down both a solution strategy and a verbal explanation of how they worked the problem, and wrote down the calculator keystrokes they used in obtaining the final answer. The teacher quickly debriefed the problem by explaining her solution strategy.

The teacher told students to get out their homework (i.e., determine whether the numbers 630, 351, 97, 3744, 1720, 61776, 22548, and 11216 are divisible by 2, 3, 4, 5, 6, 8, 9, 10). Students exchanged papers; the teacher quickly called out the correct answers while the students graded each others' work. The teacher told students to get out their spiral-bound notebooks. She wrote the word "divisible" on the board and asked a student to read aloud the definition found in the textbook glossary while she wrote the definition on the overhead. She asked the students to copy the definition from the overhead to their notebooks. This process was repeated for the terms prime number, composite number, prime factorization, and factor tree. When discussing prime numbers, the teacher made a list of primes up to 37, and, when discussing prime factorization, the teacher worked through several examples for the class (e.g., $60 = 2 \times 2 \times 3 \times 5$), forming a factor tree for each. There was no mention of alternative factor trees that might be written for a given composite number. The class ended when the bell rang, with no wrap-up or closure to the lesson.

## 7th Grade: Subtraction of Integers

The instructional materials for this lesson on subtraction of integers were taken from the new, district-adopted textbook materials. This series, which focuses heavily on drill and practice, replaced a more hands-on textbook series that the district had selected for its previous adoption. The current adoption was in response to teachers' complaints that the former adoption did not provide students with enough work on mathematics vocabulary or enough practice on important procedures.

The lesson began with a warm-up activity that included a review of adding and subtracting integers. The teacher wrote on the overhead projector as a few students volunteered answers to the teachers' questions; many students were inattentive. During this review the teacher wrote symbolic rules for adding and subtracting signed numbers (e.g., "(+) + (+) = (+)") on the overhead. Following the warm-up activity, students checked their answers to the homework assignment from a copy of answers that the teacher placed on the overhead. The teacher worked the last four homework problems for the class.

The teacher distributed three worksheets copied from resource materials that accompanied the district-adopted textbook series. The first worksheet provided practice on subtraction of integers (e.g., (-3) + 5 = __), the second was a "multicultural problem set," and the third contained a word search for mathematical terms. Students worked independently on the worksheets. At the end of the time allotted for the class, students packed up their materials and left the classroom.

## 7th Grade Pre-Algebra: Problem Solving and Inequalities

Prior to this lesson, the 7th grade pre-algebra students had worked on an extended investigation focused on multiple strategies for solving word problems. In this lesson, a new word problem was used to reinforce the need for careful reading of problems, justification of strategies used and solutions presented, and the concept that there are multiple ways to approach solving a single problem. A second focus of this lesson was on extending what students had learned previously about solving one-step equations to the solution of inequalities. The teacher stated that the content of the lesson was selected because she knew students needed to learn it in order to do well on the up-coming state assessment. The primary component of the lesson, small and large group discussion around a single word problem, was designed to have students articulate their mathematical thinking, thereby instilling both competence and confidence in their own mathematical abilities. The student make-up in this class, as per the teacher's request, represented a greater diversity than the district's and school's tracking practices usually allowed.

At the beginning of the lesson the teacher collected students' work on an extended problem-solving investigation and posed a new word problem: "Tim's father bought some baseball cards. He paid $8.00 for every six cards he bought. Later, he sold them, making a profit of $4.00 on every 3 cards. If he made a profit of $24.00 altogether, how many cards did he buy and then sell?" The teacher wrote the problem on the blackboard and students copied the problem into their mathematics notebooks. The teacher set a timer for 6 minutes and asked students to work on their own or discuss the problem with others in their group. During this time the teacher circulated among the groups, listening to students' discussions and providing encouragement. When the timer went off, the teacher asked students to share the strategy they used (e.g., picture, list, working backward) with the class. She asked a few students to present to the class, step-by-step, their solution to the problem. An open-ended, at times student-directed, discussion ensued. After about three-fourths of the class time had elapsed, the teacher stopped the discussion to move the students on to the inequalities worksheet: "I'm going to leave you to think about this more; I hope you wrote it down and can look at it tonight and ask your parents for their thoughts."

The teacher directed students to go to a box on the side counter and pick up their worksheet on inequalities from the previous day. For the remainder of the period, students worked individually or with others in their group on the inequalities worksheet. The teacher circulated around the room, answering student questions and checking on student progress. Incomplete worksheets were collected in the last few minutes of the lesson as students prepared to leave for their next class.

## 8th Grade: Percents, Decimals, and Fractions

The lesson in this 8th grade class focused on percents, decimals, and fractions. The teacher had learned about the hands-on activity incorporated in this lesson at a district-provided professional development workshop that was held two weeks earlier. The lesson was structured to allow students opportunities to talk with each other about mathematics and to engage in hands-on activities.

The lesson began with a whole-class discussion of percents, including procedures for converting among percents, decimals, and fractions. Students were asked to open their texts to a specific page and the teacher began an interactive discussion by asking students a series of questions about the meaning of "25%" and then "125%." The teacher wrote all responses on the board (e.g., 125% means 125 "out of 200," "out of 1000," "out of 100"), but moved rapidly to the correct answer. The teacher posed the problem of converting 125% into a fraction and a decimal. Students were told to work the problem on their papers as she worked it on the board. Students were then assigned three textbook problems to work on for independent practice (e.g., change 220% to a fraction, whole number, or mixed number, giving each answer in simplest form).

In the previous lesson students had begun a hands-on activity in which they used data about the way a student's budget was apportioned, per dollar, for lunch, clothing, recreation, CDs, and savings (e.g., 25 cents of every dollar was for the purchases of lunches) to create a bar graph and then turned the bar graph into a pie chart. The teacher explained that, for the rest of the period, they were to continue working on that activity. An assistant supervised the students as they worked on the activity while the teacher went to another room to teach a lesson to a different set of students. The teacher returned to the original classroom midway through their group work and again at the end. The lesson ended as the teacher collected the students' completed pie charts.

## 8th Grade: Pythagorean Theorem and Trigonometric Ratios

The 8th grade mathematics class was at the end of their study of the Pythagorean Theorem and the beginning of a study of trigonometric ratios. The teacher said that she would prefer not to deal with trigonometric ratios in 8th grade, but was doing it to teach to the state standards and thus prepare the students for the state assessment. She stated that the year before she made the students memorize the definitions of sine, cosine, and tangent, but has since realized that they are included on the formula chart passed out with the state assessment test; she now focuses instruction on making sure that students understand the terms opposite, adjacent, and hypotenuse and how they relate to the trigonometric ratios. The teacher used a non-interactive, whole-group approach for most of the lesson. A classroom rule was that there is absolutely no student talking during a mathematics lesson except to ask the teacher a question or to answer a question posed by the teacher.

The lesson began with a warm-up problem that the teacher put on the overhead and students worked on independently: "Use the figure (rectangle ABCD with an X marking the point of intersection of the two diagonals) to name an obtuse triangle." The teacher asked two students to say their answers and the teacher repeated them, briefly explaining why each answer was correct. She asked students to open their textbooks and get out their homework on the Pythagorean Theorem. She worked a few homework problems for the students, asked if there were any questions, and directed students to get out a piece of notebook paper for their journal writing that would follow the quiz. The teacher passed out the quiz sheets, and the students worked quietly and independently, putting their paper in the "in-box" as they finished and moving on to the journal-writing task. Some students finished early; other students had not begun the journal writing task when the teacher asked for papers to be turned in. The teacher told the students to make sure they had at least copied the journal assignment so they could work on it later.

The teacher distributed a handout that was a combination of worksheet and notes on trigonometric ratios. She explained, "Trigonometric ratios can be used to find the measure of one side of a right triangle if the measure of one side and the acute angles are known." The teacher demonstrated how they could solve an equation for the unknown (e.g., If $\tan 30^o = x/21$, then $x = 21 \tan 30^o$) and then asked them to do an analogous example ($\cos 45^o = x/27$). There was no exploration of what a $45^o$ angle in a right triangle implied about the other acute angle or the length of the other leg. The teacher walked around checking student progress on this problem. The teacher went through a third sample problem, reviewing what she meant by opposite and adjacent. Referring to the 9 practice problems on the worksheet, the teacher worked 3 for the class and then directed the students to complete the remaining 6 exercises.

On the overhead the teacher drew a 6-8-10 right triangle with acute angles P and Q and right angle R and she listed the prompts: (1) sin P, (2) cos P, (3) tan P, (4) sin Q, (5) cos Q, and (6) tan Q. She stated that she didn't know the value for angle P. She wrote the trigonometric ratios for the first three prompts, reducing each to a fraction in lowest terms, before being interrupted by the bell signaling the end of class.

## 8th Grade Geometry: Algebraic Multiplication and Geometric Proofs

The teacher described this 8th grade geometry course as being exactly the same as the district's high school geometry course, a course which is taught with emphasis on undefined terms, definitions, fixed sets of procedures, and formal proof. The teacher stated that she likes the textbook and that she does not depart from the structure of the course presented in the text, although she does occasionally supplement the text with problems taken from other texts. Because her students are required to take the state-mandated assessment test (which is based on the state's regular 8th grade mathematics curriculum) at the end of the year, the teacher stated that she provides daily instruction designed to help students prepare for that test alongside instruction in geometry.

The block scheduling for this class divided the period into two segments with a lunch period in the middle. For the before-lunch segment the teacher began with two warm-up problems on multiplication of algebraic fractions, e.g.:

$$\frac{10a^2 + 5a}{6m^2} \bullet \frac{18m}{4a^2 - 1}$$

Students were given time to work independently on these problems while the teacher offered assistance to individual students. The teacher then led a whole-class discussion of the solutions. The teacher returned a set of graded geometry quizzes and assigned students to pair groupings for a peer-tutoring session. Students were asked to work through the quizzes together, helping their partners as needed. After approximately 10 minutes of peer tutoring, the teacher reassembled the class and several problems from the quiz were worked on the board. Frequently, the teacher would remind students of an underlying characteristic or property that was illustrated by a given problem (e.g., reflexive property, distributive property), and she used the set of problems to illustrate differences among mathematical properties. The class continued to work on problems from the quiz until the beginning of the lunch period.

After lunch the teacher asked students to put the graded quizzes into their mathematics folders and to open their geometry textbooks to a section on geometric proof. Students were asked to read the first page quietly, after which the teacher asked if everybody understood. Hearing no questions, the teacher asked them to read on and study the proof on the next page. The lesson was devoted to the conventions of proof, so the teacher emphasized or elaborated on several points made in the text. The class continued in this way through two more textbook pages. The teacher assigned homework from this section before distributing a practice test for the state assessment. The teacher pointed out that several students were still having problems with calculation of area and perimeter, and she suggested that students memorize the formulas for calculating perimeter and area of circles. The class continued using the practice test to review for the state assessment until the period ended.

## 8th Grade Pre-Algebra: Similar Triangles and Problem-Solving Strategies

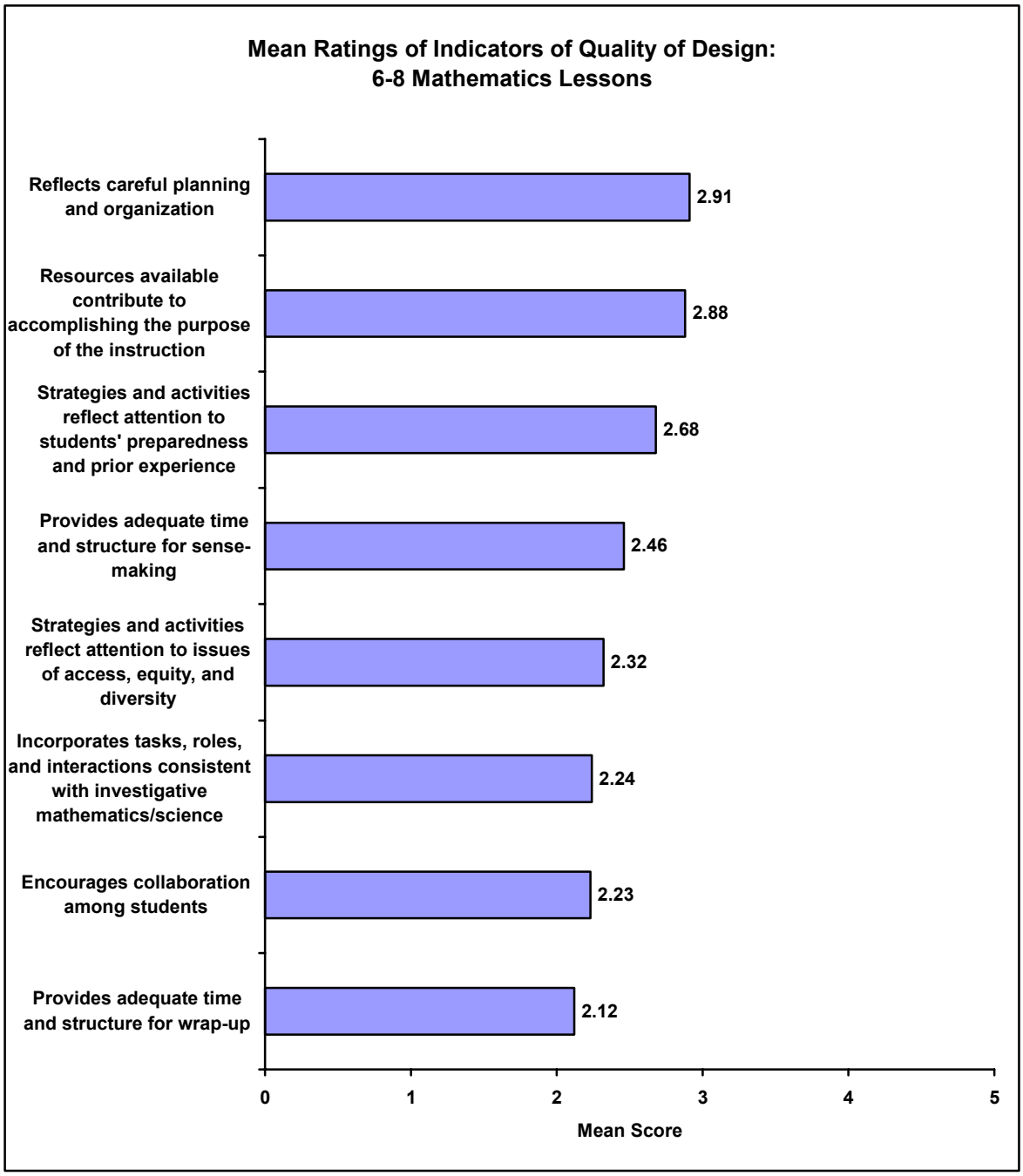
The lesson was designed to provide students with practice on similar triangles and proportions and then to move into problem-solving strategies and mathematical thinking. The class began with a review problem and a homework check and then moved into the major portion of the lesson, a problem-solving activity which was designed to prepare students for the upcoming benchmark test. The teacher described the benchmark test as being focused on problem solving and involving both multiple-choice and open-ended items. In this lesson the teacher wanted to provide an example, and a structure, for students to use when responding to open-ended problems on the test.

The teacher drew two similar triangles on the board and provided the lengths of some of the sides, asking students to determine the length of the side marked "x." Students worked alone while the teacher moved about the room to provide encouragement and answer questions. Some students began working together, and, even though the teacher did not specifically request that students talk with one another about their work, he did not seem to mind that they did so. After allowing the students a few minutes to work on the problem, the teacher led a whole-class discussion of the solution. The teacher wrote the numbers and equations on the board while the students identified and described the sequence of steps leading to a solution. The teacher asked students to check their homework (i.e., eight similar-triangle problems analogous to the warm-up problem) as he called out the answers from the teacher's edition of the textbook. The discussion turned lively when students started to debate the answers, and the teacher realized that one of the answers given in the textbook was incorrect. The teacher invited several students to the board to show how they had arrived at a different answer.

The teacher distributed two worksheets. The first one contained a word problem about mangoes, in which members of a family each take $^1/_3$ or $^1/_5$ of the mangoes in a basket until finally there are only 3 mangoes left; the task for students was to determine how many mangoes were originally in the basket. The second worksheet was for students to use to write down their solution to the problem. This worksheet contained several prompts, such as "what I know," "strategy," and "steps." The teacher asked students to carefully read the problem on the first worksheet and to answer the "what I know" question on the second worksheet by describing and paraphrasing the information given in the problem. As the students moved to the "strategy" question, some students went to a poster at the side of the room that described various problem-solving strategies (e.g., guess and check, draw a picture, work backwards). As the teacher worked with individual students, he would ask them about their strategies and answers. Throughout this segment of the lesson, students frequently discussed the problem and their thinking with each other. Work on this problem continued until the end of the class period as the students and teacher focused on processes and mathematical reasoning in problem solving. The class ended as the teacher said, "We're about out of time... Finish it for homework."

# Ratings of Lesson Components

The designs of middle school mathematics lessons are, on average, most highly rated for reflecting careful planning and organization and for utilizing the available resources to accomplish the purpose of the lesson. Middle school mathematics lessons are, on average, weak in many areas, including providing students with the time and structure needed for sense-making and wrap-up, incorporating strategies consistent with investigative mathematics, and encouraging collaboration among students. The relatively low ratings in these areas may explain why over three times as many lessons receive low synthesis ratings for their design than high ratings (58 percent and 17 percent, respectively).


Mean Ratings of Indicators of Quality of Design:
6-8 Mathematics Lessons
Reflects careful planning and organization
2.91
Resources available contribute to accomplishing the purpose of the instruction
2.88
Strategies and activities reflect attention to students' preparedness and prior experience
2.68
Provides adequate time and structure for sense-making
2.46
Strategies and activities reflect attention to issues of access, equity, and diversity
2.32
Incorporates tasks, roles, and interactions consistent with investigative mathematics/science
2.24
Encourages collaboration among students
2.23
Provides adequate time and structure for wrap-up
2.12
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1
2
3
4
5
Mean Score

*Figure D-1*

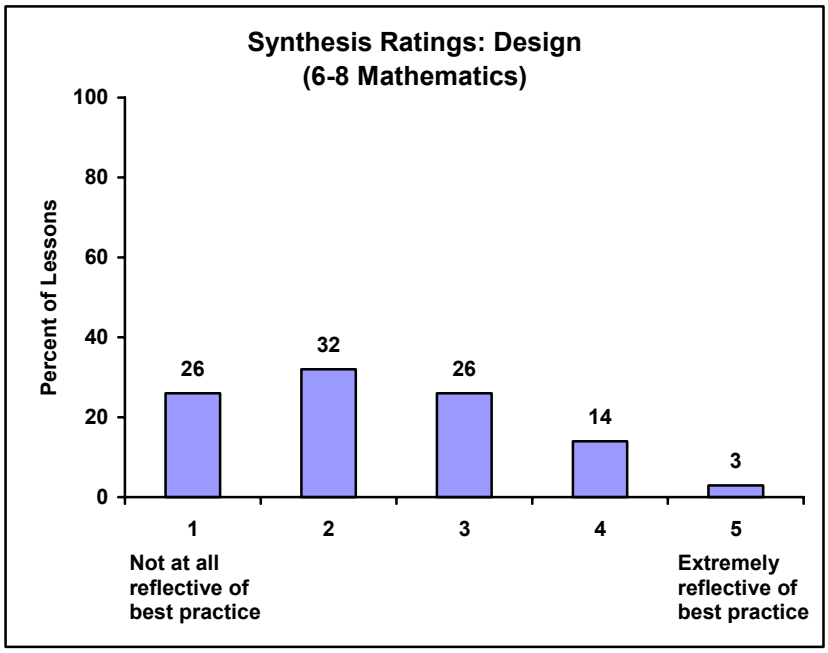


*Figure D-2*

The implementation of middle school mathematics lessons is rated most highly for teachers' confidence in their ability to teach mathematics. Lessons are weaker in regard to teachers' classroom management and pacing (moving either too quickly or too slowly). Middle school mathematics lessons are weakest in regard to adjusting instruction according to the level of student understanding, using instructional strategies consistent with investigative mathematics, and posing questions that enhance student understanding. These low ratings are reflected in the implementation synthesis ratings. Sixty-seven percent of lessons receive a low rating for implementation while only 20 percent receive a high rating.

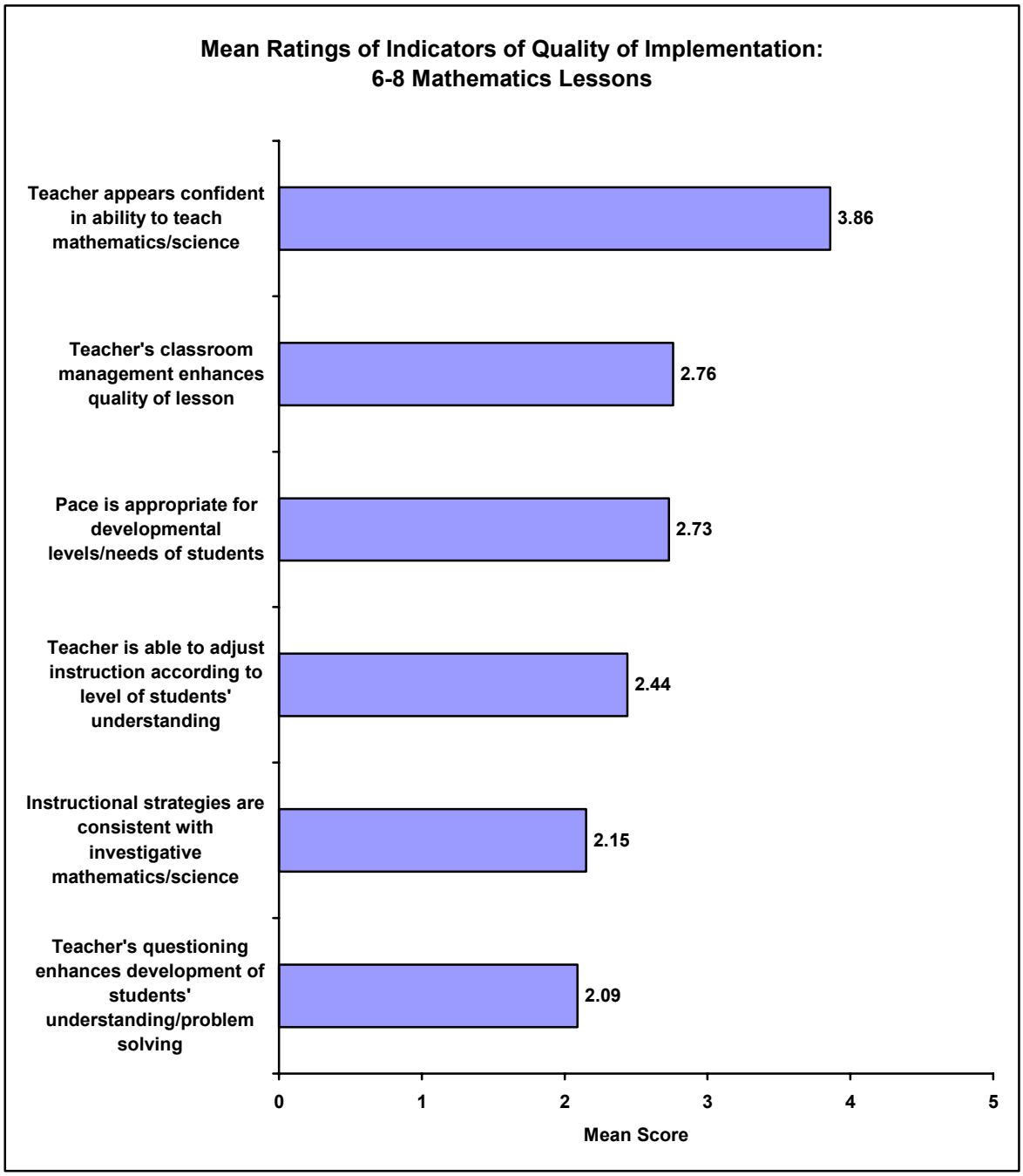
Mean Ratings of Indicators of Quality of Implementation:
6-8 Mathematics Lessons
Teacher appears confident in ability to teach mathematics/science
3.86
Teacher's classroom management enhances quality of lesson
2.76
Pace is appropriate for developmental levels/needs of students
2.73
Teacher is able to adjust instruction according to level of students' understanding
2.44
Instructional strategies are consistent with investigative mathematics/science
2.15
Teacher's questioning enhances development of students' understanding/problem solving
2.09
0
1
2
3
4
5
Mean Score

*Figure D-3*

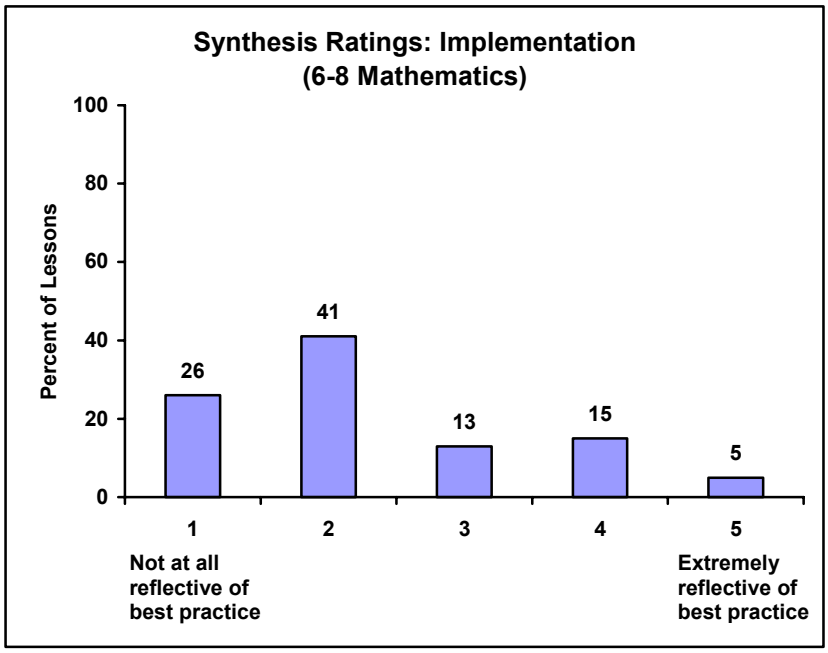


*Figure D-4*

The content of middle school mathematics lessons is, on average, rated highest for focusing on significant and worthwhile content at a developmentally appropriate level and doing so accurately. Lessons are weaker in the appropriate inclusion of abstract principles, engaging students with the content in a meaningful way, and making connections to other areas. Middle school mathematics lessons are weakest in providing opportunities for students to make sense of the content and portraying mathematics as a dynamic body of knowledge. Fifteen percent of lessons receive a high synthesis rating for content, 29 percent receive a medium rating, and 56 percent receive a low rating.

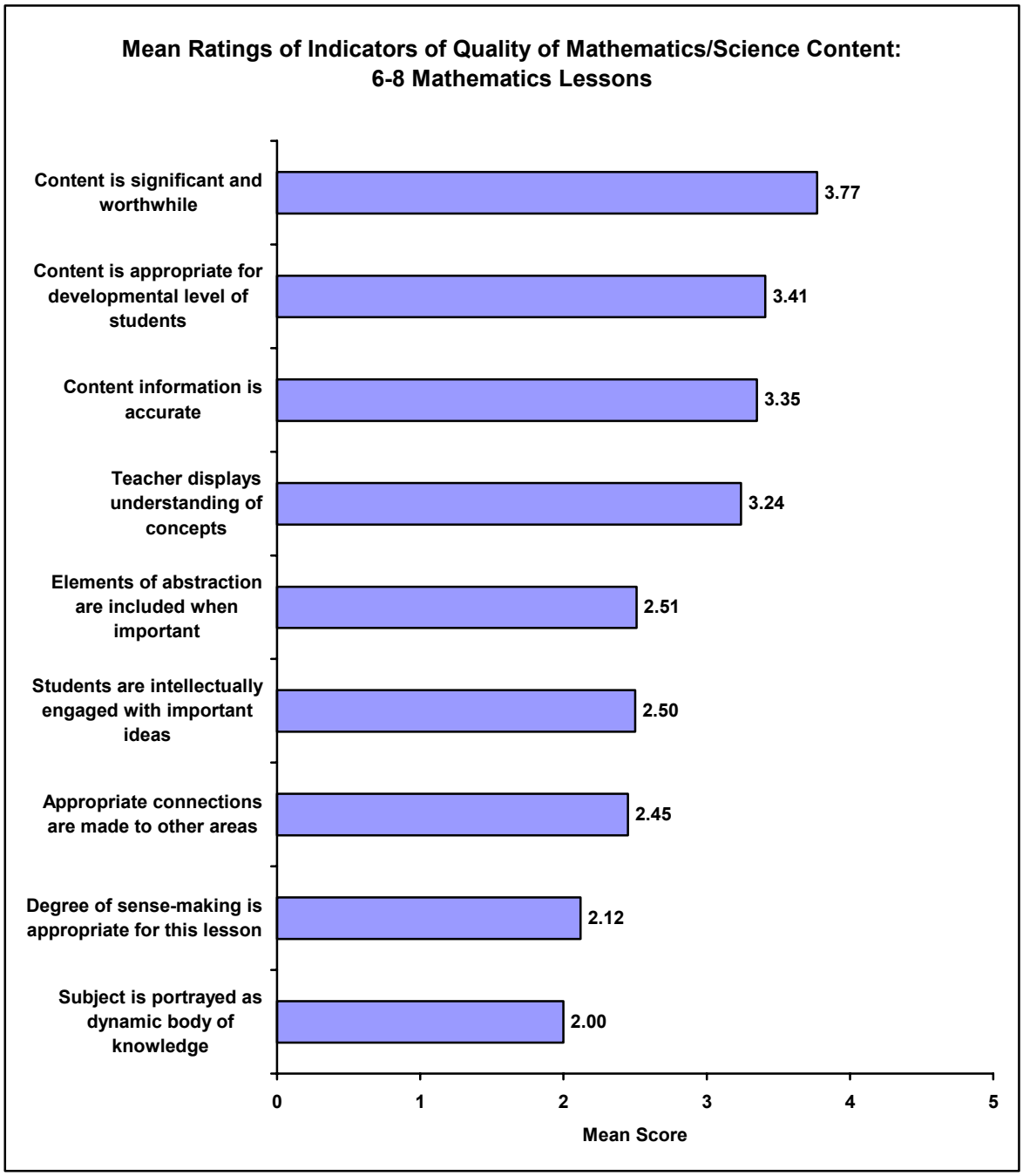
Mean Ratings of Indicators of Quality of Mathematics/Science Content:
6-8 Mathematics Lessons
Content is significant and worthwhile
3.77
Content is appropriate for developmental level of students
3.41
Content information is accurate
3.35
Teacher displays understanding of concepts
3.24
Elements of abstraction are included when important
2.51
Students are intellectually engaged with important ideas
2.50
Appropriate connections are made to other areas
2.45
Degree of sense-making is appropriate for this lesson
2.12
Subject is portrayed as dynamic body of knowledge
2.00
0
1
2
3
4
5
Mean Score

*Figure D-5*

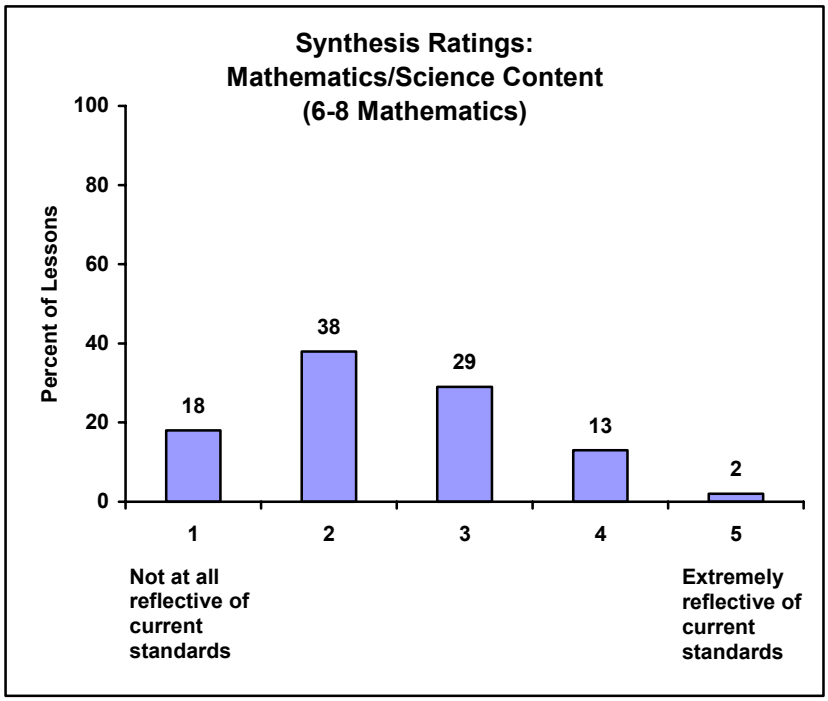


*Figure D-6*

In regard to classroom culture, middle school mathematics lessons are rated strongest for having a respectful climate and for encouraging active participation of all students. Lessons are weakest in encouraging students to generate ideas and questions and in their level of intellectual rigor. The synthesis ratings for classroom culture reflect these indicators with 25 percent of lessons receiving a high rating, 23 percent receiving a medium rating, and 53 percent receiving a low rating.

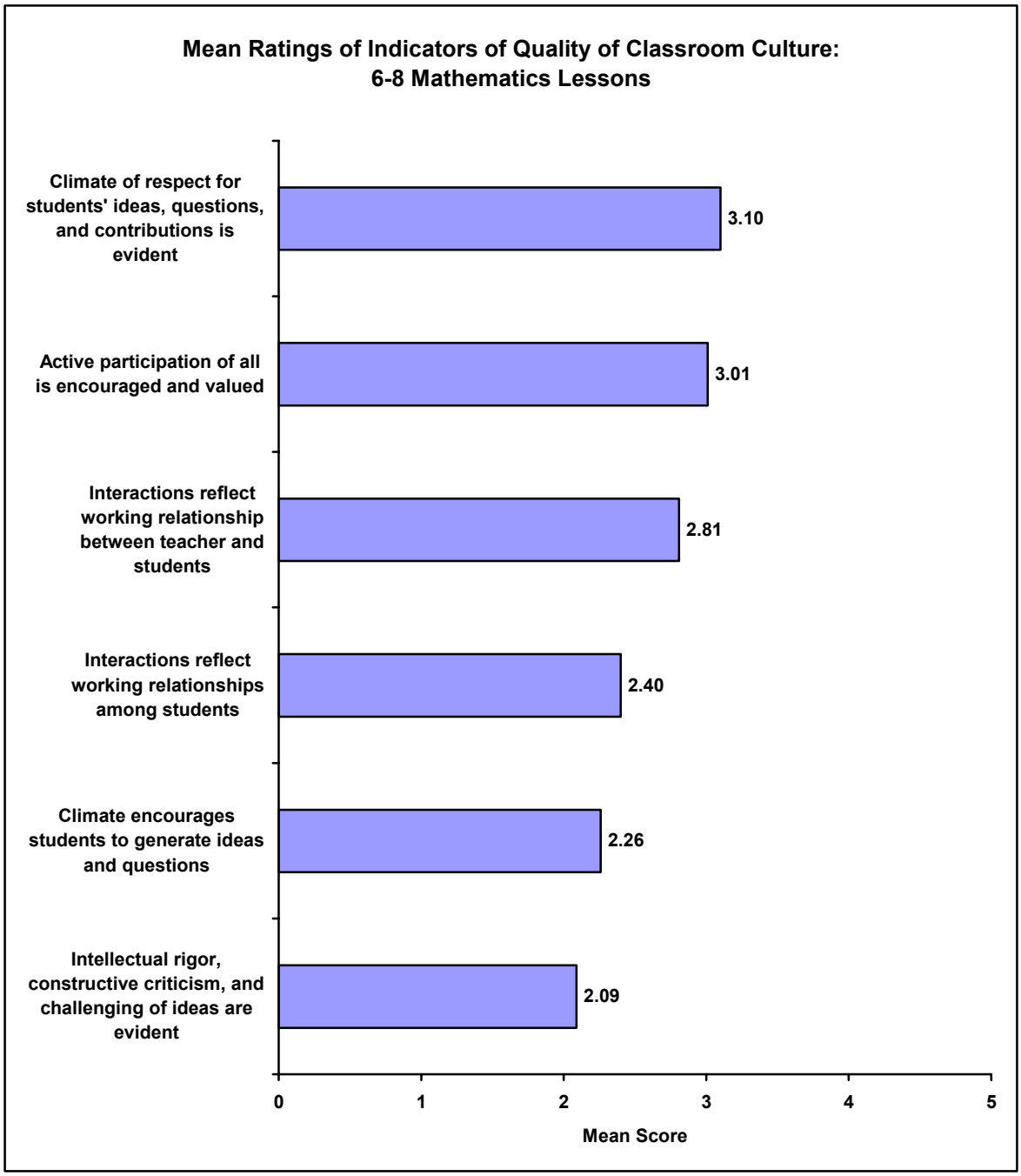
Mean Ratings of Indicators of Quality of Classroom Culture:
6-8 Mathematics Lessons
Climate of respect for students' ideas, questions, and contributions is evident
3.10
Active participation of all is encouraged and valued
3.01
Interactions reflect working relationship between teacher and students
2.81
Interactions reflect working relationships among students
2.40
Climate encourages students to generate ideas and questions
2.26
Intellectual rigor, constructive criticism, and challenging of ideas are evident
2.09
0
1
2
3
4
5
Mean Score

*Figure D-7*

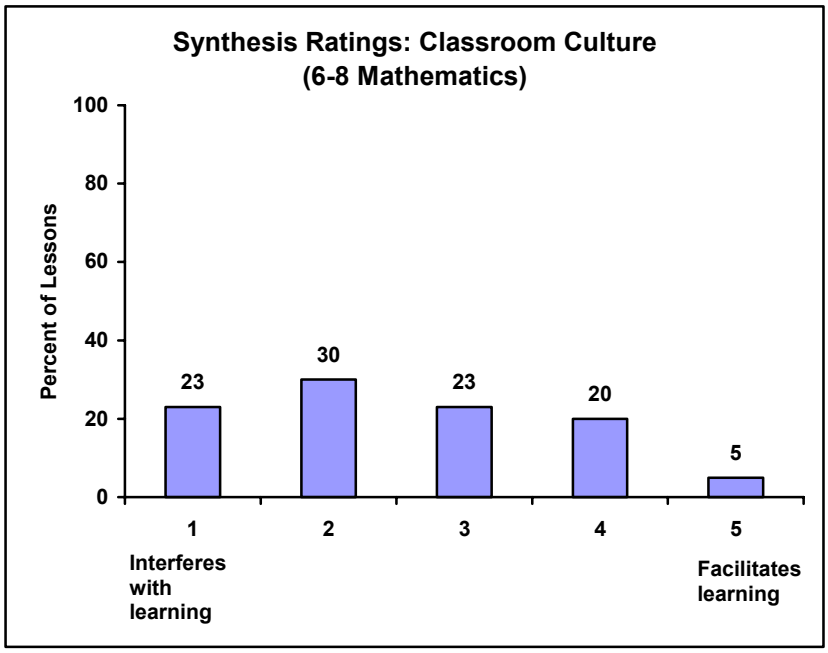
Synthesis Ratings: Classroom Culture
(6-8 Mathematics)
Percent of Lessons
100
80
60
40
20
0
23
30
23
20
5
1
2
3
4
5
Interferes with learning
Facilitates learning

*Figure D-8*

# Overall Lesson Quality

Following the ratings of the individual components of the lesson, the researcher was asked to consider the likely impacts of the lesson as a whole. Less than a third of the lessons have positive impacts on students' understanding of mathematics, confidence to do mathematics, ability to apply the skills and concepts they are learning to other disciplines or real-life situations, or interest in mathematics. (See Table D-1.)

**Table D-1**
**Likely Impact of the Lesson: 6–8 Mathematics**

| | Percent of Lessons | | |
|---|---|---|---|
| | Negative Effect | Mixed or Neutral Effect | Positive Effect |
| Students' understanding of important mathematics/science concepts | 24 | 46 | 29 |
| Students' self-confidence in doing mathematics/science | 29 | 42 | 29 |
| Students' ability to apply or generalize skills and concepts to other areas of mathematics/science, other disciplines, and/or real-life situations | 24 | 49 | 27 |
| Students' interest in and/or appreciation for the discipline | 35 | 45 | 20 |
| Students' understanding of mathematics/science as a dynamic body of knowledge generated and enriched by investigation | 49 | 33 | 18 |
| Students' capacity to carry out their own inquiries | 39 | 43 | 18 |

Figure D-9 shows the percentage of 6th–8th grade mathematics lessons in the nation rated at each of a number of levels. (See page 9 of the Observation and Analytic Protocol in Appendix A for a description of these levels.) Sixty-six percent of middle school mathematics lessons are rated as low in quality on the capsule rating, 20 percent are rated as medium in quality, and 16 percent are rated as high in quality.

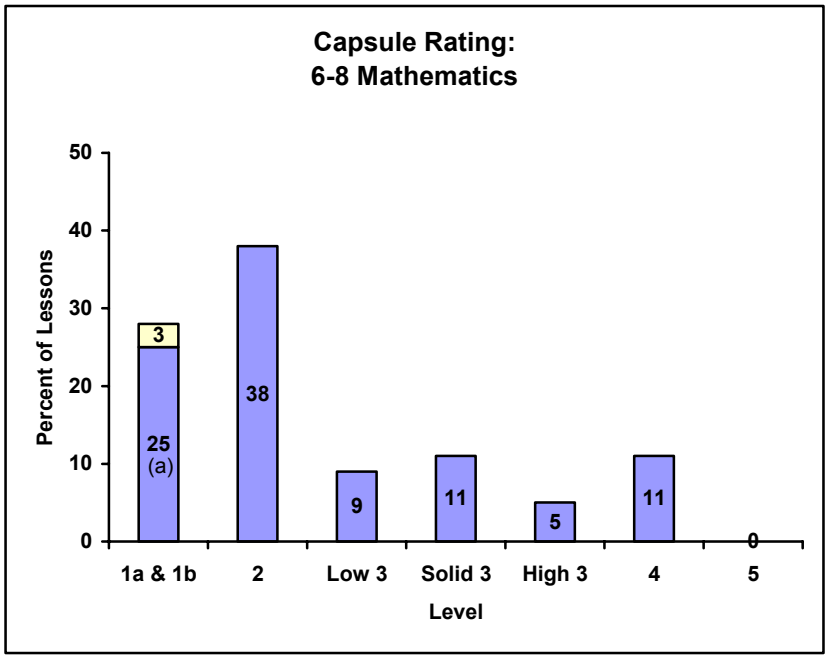


*Figure D-9*

The following illustrate lesson descriptions that were rated low, medium, and high in quality.

# Sample Low Quality Lesson: Passive "Learning"

This lesson occurred in the middle of a 6th grade unit on fractions. The class had been working on adding and subtracting fractions and mixed numbers with like denominators. According to the teacher, the purpose of the lesson was for students to learn how to find the least common denominator for a set of fractions with different denominators. Ultimately she wanted them to be comfortable enough with fractions so they would not have to consult a calculator for this work

The teacher walked the class through a couple of examples that involved addition of fractions with unlike denominators, such as $\frac{1}{4} + \frac{1}{8}$. Questions such as: "What are the first five multiples of 4?" and "What's our LCM?" and "How did I go from 4 to 8?" were typical as she explained how to do the problems. After roughly five addition examples, the teacher painstakingly repeated the process for subtraction, as if the concept was drastically different.

Following this, the teacher sent two students to the board to give the class more practice. As students worked through the problem, the teacher would probe to help them along. Upon seeing incorrect work on the board, she asked questions like, "Five times what equals ten?" and "How did you go from a 10 to a 10?…And so what's 1 times 1?"

She checked the class' understanding by asking if there were any questions. Upon receiving no questions, she passed out calculators and issued a two-page, 39-problem homework assignment for students to begin to complete individually. Although one of the pages was labeled "enrichment," it appeared to be just like the other one. Of the 39 problems, one problem involved three different fractions, but still only two different denominators. Instead of waiting to see how the students approached solving this problem, she talked through the solution.

The teacher then spent the rest of the lesson monitoring the students as they completed their work. At times she took care of logistical issues in the classroom, like organizing bookshelves. At other times she walked around and looked over students' shoulders and answered questions from individuals. As the period ended with the students working silently, the teacher collected the calculators and reminded the students to finish the rest of the work for homework.

This lesson on adding and subtracting fractions with unlike denominators was geared primarily toward having students master specific procedures without any attention toward conceptual understanding. Low-level, procedure-oriented questions dominated the teacher's discussions with the students. Poor pacing through relatively simple content and low rigor also decreased the likelihood that this lesson increased students' understanding of the content.

## Sample Low Quality Lesson: "Activity for Activity's Sake"

This 7th grade lesson occurred near the end of a unit on ratios and proportions. The lesson was designed to help students better understand how to use cross-multiplication to compare fractions and how ratios and proportions can be used in real-life situations, particularly in comparison shopping.

At the beginning of the lesson, students were responsible for completing three practice problems, all of which focused on comparing fractions. Although some students did not even attempt the problems, the teacher proceeded to briefly explain the solutions. For example, in one case he asked, "Just tell me—$3 \times 9$ and $4 \times 7$—is that equal?" Answering his own question, he stated, "No, so the fractions are not equal." In discussing the three problems, the teacher asked the students to tell him whether the fractions were "equal" or "not equal," with no attention to truly comparing fractions by determining which fraction was greater in value. Throughout the discussion, students merely had to respond with a yes or no to his questions.

The teacher then moved to introduce the main activity for the lesson by asking the students what they looked for when they shopped. This served as a poor hook because students mostly discussed brand names instead of prices. Nonetheless, the teacher transitioned to the proportion activity, asking the students to work in pairs to simulate shopping for seven school supply items. The pairs were presented prices from two different stores and asked to use proportions to determine which items would be cheaper at each store. Students were asked to determine which store would be the better choice if they only had time to shop at one and how much money they could save if they had time to go to both stores.

Throughout the activity, students selected their own ways to approach the task and to write up their work and most made use of calculators. The teacher did very little to help students focus on important mathematical ideas through his questioning or his facilitation of discussion. For example, as he worked to assist students, the teacher asked the question, "What are you supposed to do?" This as well as other questions clearly focused more on the directions for the activity and the set procedure of cross-multiplying than on the important mathematics concepts. As the activity drew to a close, the teacher asked a couple of groups to report their results to the class and then asked the students to individually write three paragraphs about their results. There was no additional discussion of ideas or approaches.

While students wrote, the teacher distributed an extra practice worksheet for homework and worked the first problem on the board. His work through the first homework problem was again completely procedural in nature and ended in a final answer that was procedurally sound, but written incorrectly as a string of equalities.

Overall, there was very little intellectual work done in this lesson. Although the content of ratio and proportion is significant for 7th grade students, the ways in which the teacher managed the content undermined the opportunity for students to develop a better understanding. The teacher primarily focused on the procedure of cross-multiplying and checking fractions for equivalence. He did very little to encourage students to make sense of how these procedures aided in determining the best prices for the school supplies. Throughout the lesson students were not asked to share ideas or justify their reasoning. The lesson did not seem to interest students and did very little to increase their knowledge of proportions.

## Sample Medium Quality Lesson: Beginning Stages of Effective Instruction

This 6th grade lesson came in the beginning of a unit designed to help students build an understanding of factors and multiples. The goal of the observed lesson was to "to reinforce what a factor is and to move into what multiples are." Prior to this lesson, students had played the "product game," and in this lesson, students were to create their own game board.

The lesson began with a discussion of the product game and the advantages and disadvantages of being the first player. They then moved on to examine the composition of the game board and discuss why some numbers were missing from the product board. Finally, the children were given an example of how to build a 3x3 game board and asked to build their own 4 x 4 board. The end of the lesson was spent with children working independently to create their own boards.

The students seemed eager to build their own game board and a few were adventurous enough (as was the intention of the design) to explore unique and different boards. The majority, however, followed the lead of the example given and used a procedure for building the board that was just like the one the teacher had given as an example. This scaffolding took away from the desired outcome of getting students intellectually engaged in an interesting problem.

The teacher's questions were very leading, and they required little thinking by the students. In fact, the majority of the students were just watching as she did all the work at the overhead projector and filled in the table and the board. In addition, the teacher was quick to hear the right answer to her questions and to move on, or did not hear the answer the way she wanted and provided the answer herself, often short-circuiting the thinking process for the students. It appeared that the teacher was not tuned into the students' understanding, until she started to circulate and watch them work on the game boards at the end of the lesson. Her focus throughout the large group discussion was on getting through the sequence of questions she had prepared.

At times there was a lack of rigor in addressing the content. For example, it was never made explicit why some prime numbers were not on the board (e.g., the prime numbers 2, 3, 5 and 7 were found on the board and 13 was not). On occasion, sloppy language, such as the student who said that 26 only had two factors 2 and 13, was not corrected or clarified. In addition, while the teacher had a number of discussions about the mathematical content with various groups of students, there was no mechanism that allowed all students to engage with all of the mathematical ideas being raised.

The lesson was rated a low 3 because there is evidence of the beginnings of effective practice. The teacher was enthusiastic about choosing a task that had the potential of being an excellent investigation of important mathematical ideas for her 6th grade students. However, the questioning strategies used by the teacher limited the effectiveness of the lesson and limited the opportunities for students to make sense of the mathematics at a deeper level of understanding.

# Sample High Quality Lesson: Traditional Instruction

This 8th grade lesson was intended to extend a brief geometry unit the teacher designed on triangles. Students had learned about right triangles and the Pythagorean Theorem. The observed lesson was the third day on the topic of similar triangles. In addition to deepening students' understanding of similar triangles, the lesson was designed to better prepare students for an open-ended benchmark assessment that was to be given soon.

Following the warm-up problem and check of homework, the teacher passed out an open-ended problem that students worked on for the rest of the class. The first page contained the problem, and the second sheet of the handout served as a recording tool for students. This sheet contained several prompts, such as "what I know," "strategy," and "steps."

After a few minutes in which students read and reflected on the problem, the teacher had students turn to the second page of the handout and begin answering the first question, which asked them to list information given in the problem that could help them solve it. Students worked independently while the teacher moved around the room and looked over shoulders. Some students went to the poster at the side of the room that described the various problem-solving strategies. The teacher also talked with one student about working backwards.

All of the questions posed by the teacher encouraged students to think about what they were doing, and challenged them to articulate their thinking with more than a one-word answer. For example, the teacher asked one student what she thought of the reasonableness of her answer. After the student responded that she thought her answer was too high, the teacher further probed to find out why the student thought that. To another student who was uncertain of his answer because it contained a decimal, the teacher asked, "What about that answer makes you think that it's unlikely?"

He seemed to provide just the right amount of support and encouragement as students struggled to solve the problem. For example, at one point he stopped the whole group to clear up an issue that was starting to be a barrier for numerous members in the class. He also drew a diagram on the board to further encourage some students to draw a diagram to help them solve the problem.

Students felt free to ask questions of the teacher and of their peers, even though the lesson did not specifically invite them to work together. The teacher allowed students to interact when it was clear they were working on the problem. At other times, students left their seats to get a closer look at some problem-solving posters hanging on the wall, and the instructor allowed this as well.

The content included in this lesson was appropriate and was presented through methods that interested and challenged the students. The lesson did a fine job of combining test-preparation and a review of problem-solving strategies. Although the pieces were not a perfect fit, the culture of the class and the investigative quality of the sections pulled the class together. The students remained engaged throughout the class, with the teacher clarifying and focusing their efforts when needed. The lesson was highly likely to extend students knowledge of similar triangles as well as better equip students to conduct their own mathematics investigations.

# Sample High Quality Lesson: Reform-Oriented Instruction

The purpose of this 7th grade pre-algebra lesson was to help students better understand the measurement of interior and exterior angles of polygons. Students were in the middle of a geometry unit and had already learned about angles, triangles, and quadrilaterals.

The class began with students completing and discussing a starter problem from a book of mathematics Olympiad exercises. After allowing students to talk through their approach to the problem, the teacher shifted gears and gave answers to the homework from the previous day. She then began a whole-group discussion on the new material.

The teacher's skill as a facilitator enabled the whole-group discussion to develop into a rich, discovery-based lesson. As the teacher asked probing questions, students were willing and eager to share ideas. At times, the teacher directed specific questions to the few students who were not so quick to respond.

To assist students in making sense of the content, the teacher used several polygons as examples. As she drew on the overhead, students followed along by recording ideas in their learning logs. The teacher frequently invited peer criticism for students' work. At their seats, students were instructed to consult one another and to check their neighbors' work. In addition, the teacher solicited students' thoughts on work others presented in the class. For example, after one student displayed her solution to a problem, the teacher asked, "How many got the same answer? Did anyone use a different approach?" After another student volunteered to come up to the board to show another strategy, the teacher then remarked, "Interesting. How many students got that for an answer? Are there other answers different than either of these? Which makes more sense?"

The teacher also incorporated several techniques for gauging the understanding of the group. For example, the class was asked to give feedback at several critical points in the discussion. "Thumbs up if you agree, down if you disagree, and sideways if you aren't sure," she stated periodically during the lesson. Students appeared very comfortable sharing their thoughts and uncertainties.

Throughout the discussion, the teacher continued to question students to help them better understand the concepts. In one case, after an extended period of discussion and much anticipation, she asked the students for a drum roll right before she divulged the correct answer to the mathematical issue.

The overall implementation and classroom culture of this lesson were outstanding. Students were intellectually engaged in meaningful mathematical content that was developmentally appropriate for a pre-algebra course. The teacher gave frequent praise and encouragement, and regularly checked for student understanding. She also provided ample opportunity for students to make sense of the content, as well as challenge the thoughts of their peers. It is quite likely that students were able to leave the lesson with a deeper understanding of interior and exterior angles of polygons.