Appendix C

Grades K–5 Mathematics

Typical Lessons

Ratings of Lesson Components

Overall Lesson Quality

Typical Lessons

The following lesson descriptions are based on a random sample of K–5th grade mathematics lessons.

Kindergarten: Time, Patterning, and Counting-on

Several content topics from the district's curriculum guide were included in this kindergarten lesson as the teacher strived to push students' thinking in ways that she felt would better prepare them for increasingly difficult state assessment tests. Activities based on telling time, creating patterns with blocks, and addition of whole numbers using a counting-on strategy were included along with a routine daily mathematics review of the days of the week, before and after, and place value.

The lesson began with an activity on time in which students were asked to show specified times using small paper clocks with movable hands. The daily mathematics review activity was next. Referring to a calendar that was hanging on the wall, the teacher asked students to name the day of the week for today, yesterday, and tomorrow and to say the day's date in unison. The class did a pattern of claps and snaps in unison as they went through the current month, giving a clap for every colored heart (days in school) and a snap for every clear one on the calendar (days not in school). The class then used straws and a place value chart to show the number of days they had been in school since the beginning of the year.

Working individually, students created patterns using whatever objects they chose from the manipulatives bin (e.g., bears, blocks, legos). The teacher then called for students to join her on the carpet. Together the students acted out an addition counting-on strategy that they had learned in an earlier lesson (i.e., 8 + 3 = ___ was acted out with "8 on the head and then 3 fingers up" and counting-on from 8). After they completed the addition activity, the teacher asked students to read a few sentences off a wall hanging, thereby signaling the end of the mathematics lesson.

Kindergarten: Comparing Lengths of Objects

This lesson focused on comparisons of lengths of objects and development of vocabulary related to comparisons (e.g., longest, shortest, between, order). The teacher supplemented the lesson from the textbook by reading the class a story related to this topic.

The students, sitting around the teacher, were led through several short activities that all involved comparisons of lengths of different objects: strips of paper, student height, and straws. For example, the teacher asked the students to look at three different lengths of paper strips and think about placing them in order by length. She asked the class to tell her which strip was the shortest, which was next, and which was the longest. The students called out the colors of the strips in order of increasing length. This same process was repeated as the teacher asked students of different heights to stand next to her. Following these two activities, the teacher read aloud *The Biggest Fish* by Sheila Keenan. In the book people from a small town were trying to make their town famous by catching the biggest fish. As the story progressed, each new person came with a bigger fish than the previous person. The children listened attentively as the teacher read the story.

Returning to the text activities, every student was given a bag containing an identical set of four straws of different lengths. The teacher asked them to decide which was the biggest or longest straw and to name its color. This was repeated until the straws were ordered from longest to shortest. The teacher asked questions focused on comparing the different lengths, "I want you to show me a straw that is shorter than your red straw and longer than your yellow straw and hold it up." The lesson ended as the teacher reminded the students of an activity in a previous lesson in which they had measured each other's feet and ordered them from shortest to longest and asked them to recall who had the longest foot and who had the shortest foot.

1st Grade: Addition Patterns

This 1st grade lesson on doubles occurred after the class had studied addition and subtraction facts to sum 12. The teacher stated that she adheres to the textbook ordering of topics, even though she thinks the text skips around a lot. She also references her lesson objectives to the state-mandated mathematics curriculum. Materials for this lesson came from the text and its supporting materials. Specifically, the teacher described the purpose of the lesson as teaching doubles facts and helping students see patterns in adding.

After a warm-up activity in which students were asked about the number of muffins that would fit in two, six-cup muffin pans, students were instructed to use a die to generate double facts. Every student rolled a die to get a number, wrote that number down, added the same number to it, and found the answer (e.g., if a 4 was rolled on the die, the number sentence 4 + 4 = 8 was written). For each of the six possible double facts, the teacher selected a student to write the equation on the board and to read the equation aloud to the class.

The teacher distributed a textbook enrichment activity worksheet containing six word problems (e.g., "My double is between 5 and 7. What number am I?" and "The parrot picks six nuts. Then he picks 6 more. How many nuts does he pick?"). The teacher read each problem aloud and worked it for the students. Students were given another worksheet to complete as independent practice and to place in their folders when finished. The final activity in this lesson was a game of math BINGO in which BINGO cards had an addition fact problem in each square (e.g., 5 + 5 = ___); students covered a square on the BINGO card as the matching sum was called.

1st Grade: Creating Bar Graphs

The purpose of this fourth lesson in a week-long unit on graphing for 1st graders was to provide students with practice activities related to graphing, including using tally marks as a record-keeping method and interpreting graphs. The teacher selected these topics for the lesson for three reasons: they are part of the district curriculum, they are included in the district's adopted mathematics program, and the teacher feels that graphing provides a nice change for students since it follows "harder units" on addition and subtraction. Although the teacher felt pressured by the district to use the textbook materials as prescribed in the teachers' manual, she did augment the lesson with a whole-class graphing activity before assigning the textbook-designated individual task.

The teacher modeled the graphing activities for the whole class. Students were asked the question, "What do you like to draw?" The teacher used tally marks to record students' responses and then asked a series of questions (e.g., "Which picture is drawn the most?" and "Which had less—ducks or fish?"). Next the teacher helped students form "human graphs" based on attributes of hair color, eye color, and favorite weekend activities. Students were then asked a series of questions that required them to interpret the graphs they created.

After practicing these graphing activities, the students returned to their desks and began working on the textbook-defined individual activity. In this activity students were asked to record the names of six pre-selected activities in the first column of the table on the worksheet and then to tally the responses from their classmates in the second column. The students were then asked to translate the tally marks into a bar graph, showing only the four activities with the most tally marks, and to answer questions based on their bar graphs (e.g., which activity was liked most, which activity was liked least, what would happen to the graph if building a snowman was added to the list). During the last segment of the lesson, groups of students rotated through five mathematics centers in order to practice skills, including addition facts, place value, and number recognition, using computer software and manipulatives.

2nd Grade: Addition of Two-Digit Numbers

Two sets of mathematics instructional materials were being used in this classroom—a traditional textbook series and specific units from an investigative mathematics series that was being piloted in the district. Prior to this lesson, the 2nd grade class had completed the addition/subtraction unit from the set of investigative materials. As part of the unit, students had invented a variety of algorithms for adding and subtracting two- and three-digit numbers. Students' poor standardized test results following the teaching of that unit led the school's 2nd grade teachers to decide that they should teach the traditional algorithms. The purpose of this lesson was to teach students the traditional addition algorithm and to provide students with the opportunity to practice the algorithm in a game setting.

The lesson began with a "Problem of the Day" in which students were asked to reason about how many apples were in a mystery bag on one side of a see-saw given information about what was on the other side; this problem was unrelated to the major content focus of the lesson. The students seemed to enjoy having the opportunity to explain the reasoning processes they used in solving the problem.

The teacher introduced students to a game from the traditional textbook series, "That's Sum Toss," and using the overhead projector she modeled a complete game. In the game, each player was to roll a pair of dice, write the larger of the two, two-digit numbers that could be written with the numbers represented on the top faces of the dice (e.g., 41 rather than 14), repeat with a second roll, and use the traditional addition algorithm to add the two numbers. Working in table groups, players were instructed to generate their own numbers and to keep a running total until they reached the total sum of seven, two-digit numbers. The object of the game was to be the player with the highest total sum. As the teacher modeled a complete game, she demonstrated the traditional algorithm for each sum. The teacher emphasized that students must start by adding the numbers in the ones place and move to the left. No mention was made of how the traditional algorithm related to algorithms the students had invented as they had done the investigative mathematics unit.

Initially, as they played the game in table groups, students either worked together on an addition problem or a student would do an addition problem for another student. Students at one table used calculators rather than the algorithm. As students finished getting their total sums, they worked independently on the exercises printed on the back side of the game recording sheet and, as time allowed, completed other worksheets that had been distributed during an earlier lesson. Students who were struggling with the algorithm (approximately one-third of the class) were then left to complete the game by themselves. When the teacher called time, two students were still playing the game and a few students had completed everything and were working on an assignment for another subject. The teacher ended the lesson by asking who had gotten the highest game total and by doing a quick discussion of the "challenge problem" on the back of the game recording sheet ("Find a number between 121 and 125 which, when you add 100 to it, results in a number with all three digits the same.").

2nd Grade: Subtraction Facts

The lesson, which occurred near the end of a 2nd grade unit on subtraction, was intended to extend students' knowledge of and facility with subtraction facts. It addressed two practice worksheets from the textbook resource materials that had been assigned for homework during the previous class. The first worksheet directed students to solve two word problems by using pictures as a solution strategy (e.g., "Use the number card pictures to solve the problem. Blaine had a birthday. How old was he? You say the number when you count by 3s. It is greater than the difference between 16 and 9. It is less than the difference between 12 and 1."). The second worksheet asked students to write an addition or subtraction number to go with each of six situations (e.g., "Cornelius has 13 crackers. He eats 7. How many crackers does he have now?"). The teacher began this lesson by asking students which homework problems gave them the most trouble. The problem that caused trouble for most of the students was the "Blaine's age" problem from the first worksheet. The teacher gave a step-by-step explanation of the solution to this problem and then she collected the homework.

The teacher did a brief introduction to the topic of subtracting numbers up to fourteen and gave students two minutes to think of as many number expressions as they could that equal fourteen. She listed the number expressions on the board as the students said them aloud. She asked the class questions about the number expressions they wrote (e.g., "How did you come up with 23 – 9?" and "How could you change 23 – 9 to get 25 – 11?"). The teacher asked students to open their mathematics workbooks to two pages that contained twenty-three exercises, seventeen of which were basic addition and subtraction fact problems (e.g., 14 – 9; 5 + 6). The teacher demonstrated how the first two exercises should be solved using the solution method presented in the textbook. Giving them the option of working individually or in pairs, the teacher instructed the students to complete the two workbook pages; most of the students chose to work alone. Students were allowed to use cubes and "Touch Math" to determine answers. The teacher circulated around the room, observing students' work but interacting with students only when a student asked her a question. Students who finished early were not given anything else to do. The lesson ended as the teacher began a lesson in another subject area.

3rd Grade: Fractions

This 3rd grade lesson on fractions was based on a lesson in the district-designated textbook series. The teacher taught the lesson as designed, alternating between whole class discussions/demonstrations and tasks which students completed independently.

To begin the lesson, the teacher asked each student to scoop up two handfuls of pattern blocks from a large bin. After the students had played freely with the blocks for ten minutes (creating large and sophisticated arrangements and complex sequences of colors and shapes), the teacher asked the students to separate the yellow, red, blue, and green blocks from the rest of the blocks and to put the other blocks back into the original pattern block bin. Using the textbook lesson script, the teacher led the students through an exploration of the fractional relationships of the red, blue, and green pieces (parts) relative to the yellow piece (whole). Throughout this segment of the lesson, the teacher talked about the colors of the blocks but not the shapes (e.g., ½ of yellow is red; $^1/_3$ of yellow is blue; $^1/_6$ of yellow is green). At the end of the exploration the teacher summarized their findings, "The red is ½, the blue is $^1/_3$, the green is $^1/_6$," without any discussion of the "whole." Next the teacher asked students to hold up pattern block pieces that show 1½ (1 yellow and 1 red), 2½ (2 yellow and 1 red), and 3½ (3 yellow and 1 red).

The teacher distributed a worksheet that had eight separate hexagons drawn on it; each hexagon was the same size as a yellow pattern block piece. Students were assigned the task of covering each hexagon on the worksheet with any combination of red, blue, and green blocks. For each arrangement, they were asked to (a) trace the blocks used in covering the hexagon and to color each region the same color as the block that had covered the region and (b) write a number sentence corresponding to the arrangement. The teacher demonstrated the task on the overhead by placing one red, one green, and one blue block on a hexagon and writing the number sentence $\frac{1}{2} + {}^1/_6 + {}^1/_3 = 1$. Students worked individually on this task until the end of the class.

3rd–4th Grade: Long Division

The lesson for this class of 3rd and 4th graders focused on introducing students to the traditional long division algorithm and to related vocabulary (i.e., divisor, dividend, quotient, and remainder). The teacher taught this lesson because division with remainders was a content topic found on the district's list of core concepts. The design of this lesson was based on the teacher's belief that manipulatives are an important component of instruction, and her perception that students need to have a lot of experience with mathematical procedures.

The teacher began the lesson by giving each student in the class a bag of M&Ms. Together the class modeled five problems using the M&Ms (e.g., "Count out 29 M&Ms. How many groups of 4 can you make?"). For each problem the teacher demonstrated the traditional long division algorithm, step-by-step, emphasizing the sequence of steps—divide, multiply, subtract.

The teacher told the students to put the M&Ms away, and she gave each student a worksheet and a multiplication basic facts table. The teacher showed how the multiplication table could be used to get the quotient for a division problem. The class worked the first three problems on the worksheet together, with the teacher asking the same set of questions for each problem (e.g., "Where do we put the divisor?" and "What do we do next?"). When one student found the remainder first (i.e., subtracted the closest multiple of the divisor from the dividend first, then determined the quotient), the teacher quickly told him that he was skipping the "divide" step in the divide/multiply/subtract procedure. When several students tried to use mental strategies, the teacher insisted that they adhere strictly to the designated sequence of steps. The teacher assigned the remainder of the worksheet problems for independent practice.

4th Grade: Creating Bar Graphs

This 4th grade lesson on graphing was a continuation of a textbook activity begun the previous day. The teacher modified the textbook-described lesson in ways that she thought would make the lesson more appropriate for her class of low-achieving students. She simplified the procedure so that students would be constructing single bar graphs instead of double bar graphs, and, because she believes that computer use is an important skill for students, she added a computer-graphing component to the textbook lesson.

The teacher began the lesson with two "problems of the day" (e.g., "My product is 48. If I add them it will be 14. If I subtract them it will be 2. Who am I?"). Students worked independently on the two problems while the teacher gave them words of encouragement but no assistance in obtaining answers. As students successfully completed the problems, the teacher rewarded them with praise and candy.

Each student was then instructed to return to the activity began during the previous lesson—starting from where they were, individually, at the end of that class. In this activity students worked independently to collect data (i.e., count how may drops of water could fit on a penny, nickel, dime, quarter, half-dollar, and dollar coin), to enter the data into an Excel spreadsheet and produce a bar graph, and to construct a bar graph by hand on a large piece of chart paper. Throughout the lesson the teacher's attention was on keeping students focused and providing students with instructions on what to do next. Although students were at different phases of the activity, the teacher decided to call them together midway through the class to give instructions for making bar graphs by hand. She showed them how to set up the graphs, recommending but not requiring a vertical interval of 10. She drew a sample graph on the overhead projector to demonstrate how to label a bar graph. At this point in the lesson, two students were still collecting data, two were working on computers, and four were working on their by-hand graphs. Near the end of the class period the teacher engaged students in a discussion that focused on reasons why the data varied (e.g., why one student got 23 drops of water on a penny and another student was able to get 26 drops on it). The teacher ended the lesson by saying, "Finish up what you're doing. We'll look at what you've done and then work on line graphs tomorrow."

4th–5th Grade: Constructing Rectangles with Specific Areas

The teacher indicated that the purpose of this lesson was "to help students understand what area is and to also reinforce the formula of how to find the area." More specifically, this lesson for 4th and 5th grade low-achieving students was designed to show students how they can construct rectangles that have specified areas. The teacher stated that she chose this content for the lesson because (a) it is listed in the state mathematics standards, (b) she and the two other teachers at her grade level, collectively, planned for these two weeks of instruction on area, and (c) she felt that her students needed more practice with finding areas of rectangles and that they especially needed a more hands-on approach than she had used in introducing the concept in the previous lesson. (In the previous lesson the teacher introduced the concept of area using worksheets and she "went over" the formula for calculating areas of rectangles.) The teacher stated that she never uses the designated textbook because she prefers to select activities from her collection of resource materials.

As a warm-up activity, each student was given a sheet of paper and asked to complete the four number sentences shown on the overhead (98 – 39 = ___; 80 – 59 = ___; 8 × 4 = ___; and 56 × 3 = ___). The teacher collected the students' papers and graded them before continuing with the lesson.

The teacher then led a brief whole-class discussion that was designed to recap the previous lesson on areas of rectangles. During this discussion, students talked about "counting boxes inside rectangles" and using the length and width of rectangles to find areas. Next the teacher described the activity she wanted the students to work on for the remainder of the lesson; the worksheet used in this activity was copied from one of the teacher's many resource books. Students were told to use centimeter grid paper, scissors, and crayons for the activity. Their task was to cut out 20 squares (1 cm x 1 cm), color each square, and glue squares on a piece of construction paper to form two different rectangles. Students were told that they could make rectangles having areas of 8 cm^2, 12 cm^2, 16 cm^2, 18 cm^2, or 20 cm^2. The teacher explained what 1 cm by 6 cm and 2 cm by 3 cm rectangles would look like; she modeled these two rectangles on construction paper and hung the model beside the overhead projector to serve as an example of what the students were being asked to do. Students worked independently on the task for the remainder of the class period. Two minutes before the end of class only a few students had formed rectangles with their squares and several students were still cutting out squares. The teacher told the students to put all the materials away and return to their respective classrooms.

5th Grade: Improper Fractions and Mixed Numbers

The teacher stated that the purpose of this lesson for 5th grade low-performing students was "to make sure students could find equivalent fractions and change improper fractions to mixed numbers and vice versa." He considered this mathematics content to be important because, according to the state curriculum framework, this content is to be mastered by all 5th grade students and because the state's 5th grade mathematics test contains many items on fractions. The design of the lesson was based on the teacher's belief that students need to review skills and facts daily and that an oral presentation of material is the most meaningful way to teach the children.

The lesson began with a 40-minute, whole-class, mixed review. During this segment of the lesson, the teacher asked questions about the metric system; the meaning of base 10; place value; multiplication; division; fractions; decimals; mixed numbers; improper fractions; fraction names for one; equivalent fractions; simplifying fractions; divisibility rules for 2, 3, 5, and 10; writing numbers in base 5 and base 3; place value in these two bases; changing mixed numbers to improper fractions; defining fractions as division; real-world occupations that use fractions; comparing fractions using cross multiplication, common denominators; and changing a fraction to a decimal and then to a percent. The review was almost totally an oral review. Students had no paper, pencils, worksheets, or text, and the teacher wrote very little on the board. The same group of students repeatedly volunteered answers while the rest of the students were quiet.

For the remainder of the lesson students worked independently on four worksheet pages; the problems on these pages required that students convert mixed numbers to improper fractions and convert improper fractions to mixed numbers. As the students worked on the assignment, the teacher worked at his desk unless a student asked a question. When one student asked the teacher for help in converting $1^1/_3$ to a mixed number, the teacher responded with a series of instructions (i.e., "What division problem does $1^1/_3$ represent?... No, switch the numbers around, read from top to bottom... Okay, now write the division problem... Now, do the division."); the teacher pulled out a bag of chips and told the student to use the chips to do the division. Students continued to work on the worksheets until the class ended.

5th Grade: Problem-Solving Strategies

This was a stand-alone lesson on problem solving that was embedded in a textbook unit on multiplication of decimals. The purpose of this lesson was to give the class of 5th graders opportunities to apply previously learned problem-solving strategies to a set of textbook word problems. The teacher admitted that she did not like interrupting the flow of the syllabus to teach a lesson on problem solving, but she did so because that was what was in the textbook. The pedagogy she implemented was based on her belief that students work better in groups than on their own.

The lesson began with a teacher-led discussion of problem-solving strategies that the class had used in previous lessons. Students named several strategies (i.e., making charts, making tables, estimating, working backward, and guess and check). The teacher worked through a problem example with the students, using charting as the selected problem-solving strategy, and then divided the class into groups of three and assigned each group one problem from the set of problems in the textbook lesson. Students were told that they would be asked to explain the solution to the problem to the large group and that each member of the group should be prepared to answer any questions the large group might ask about the solution. While students worked on their assigned problem, the teacher circulated among the groups. When a group seemed stalled, she asked questions to redirect their thinking. Sometimes the teacher's question pointed out a piece of information the group had failed to include in its thinking (e.g., "How many feet are in a yard?"). At other times, the teacher's question asked the students to think about the problem in a different way (e.g., "Have you considered using 'guess and check' to solve this problem?").

When the groups had finished, each group took a turn at the blackboard describing the problem situation and displaying their solution on the board. Students at their seats asked questions about the displayed solutions and also copied the work from the board into their notebooks. At the end of the presentations, the teacher asked the students to consider all the different problem-solving strategies the groups had used. A student asked a question about how to choose a good way to solve a problem. The class discussed this for a few minutes but did not arrive at a definitive answer.

In the last segment of the lesson, students were assigned a group of textbook practice problems related to the multiplication of decimals. They were given the choice of working together or working alone; individuals and groups of students worked on the assigned problems until the end of the class period.

Ratings of Lesson Components

The designs of elementary mathematics lessons are, on average, most highly rated for utilizing the available resources to accomplish the purpose of the lesson and for reflecting careful planning and organization. Elementary mathematics lessons are rated lowest in encouraging collaboration among students and for providing students with the time and structure needed for sense-making and wrap-up. The relatively low ratings in these areas may explain why over twice as many lessons receive low synthesis ratings for their design than high ratings (39 percent and 18 percent, respectively).

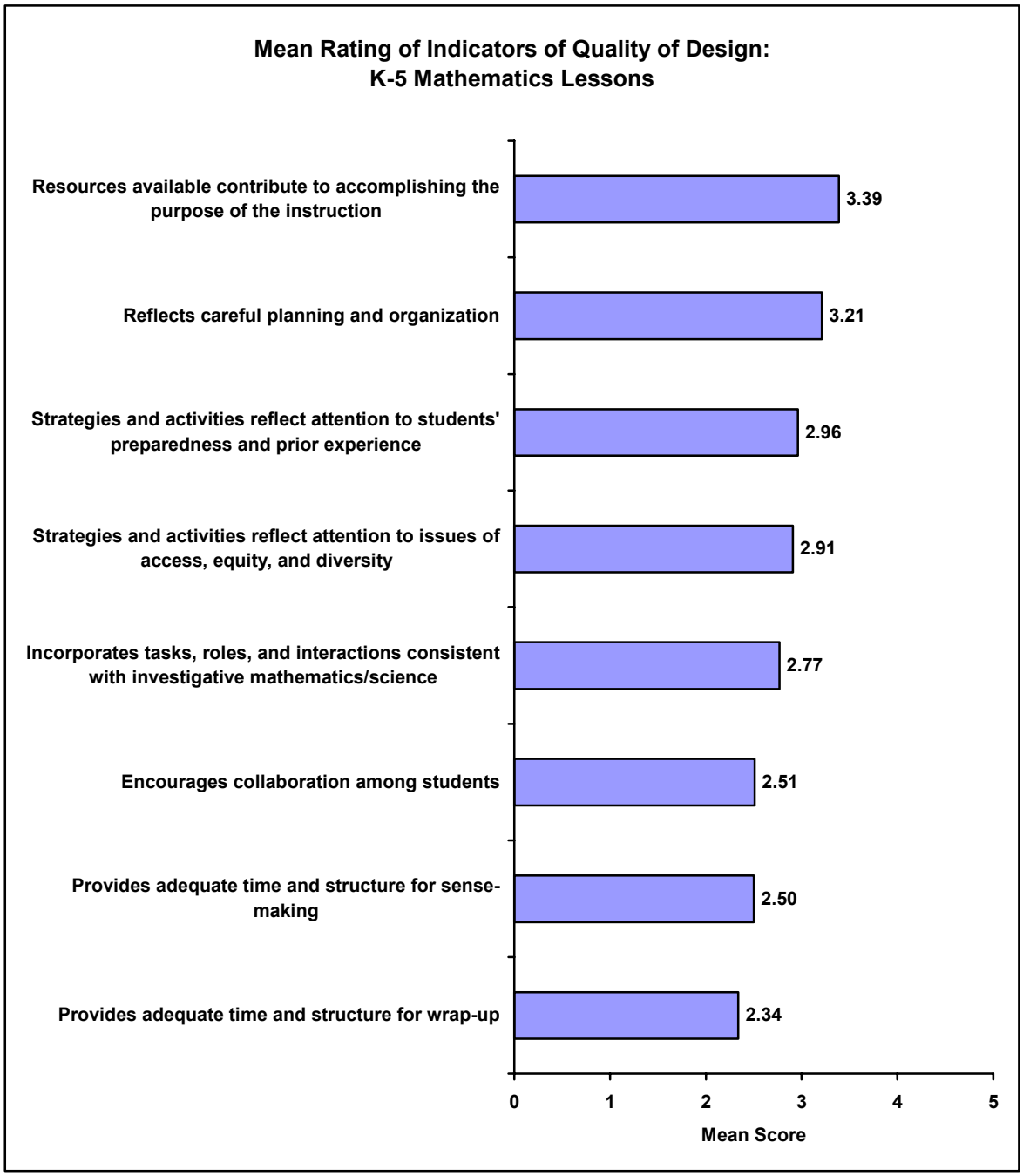
Mean Rating of Indicators of Quality of Design:
K-5 Mathematics Lessons
Resources available contribute to accomplishing the purpose of the instruction
3.39
Reflects careful planning and organization
3.21
Strategies and activities reflect attention to students' preparedness and prior experience
2.96
Strategies and activities reflect attention to issues of access, equity, and diversity
2.91
Incorporates tasks, roles, and interactions consistent with investigative mathematics/science
2.77
Encourages collaboration among students
2.51
Provides adequate time and structure for sense-making
2.50
Provides adequate time and structure for wrap-up
2.34
0
1
2
3
4
5
Mean Score

Figure C-1

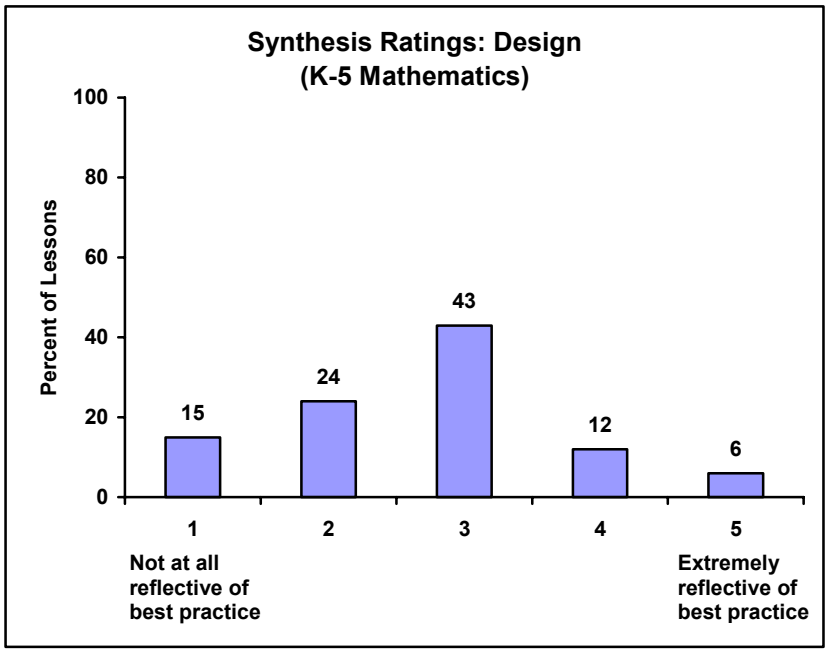

Figure C-2

The implementation of elementary mathematics lessons is rated most highly for teachers' confidence in their ability to teach mathematics. Lessons are relatively strong in regards to teachers' classroom management as well. However, lessons are weaker in terms of pacing (moving either too quickly or too slowly), using instructional strategies consistent with investigative mathematics, and adjusting instruction according to the level of student understanding. Asking questions that enhance student understanding is the weakest element of elementary mathematics lessons. These low ratings are reflected in the implementation synthesis ratings. Forty-six percent of lessons receive a low rating for implementation, while only 17 percent receive a high rating.

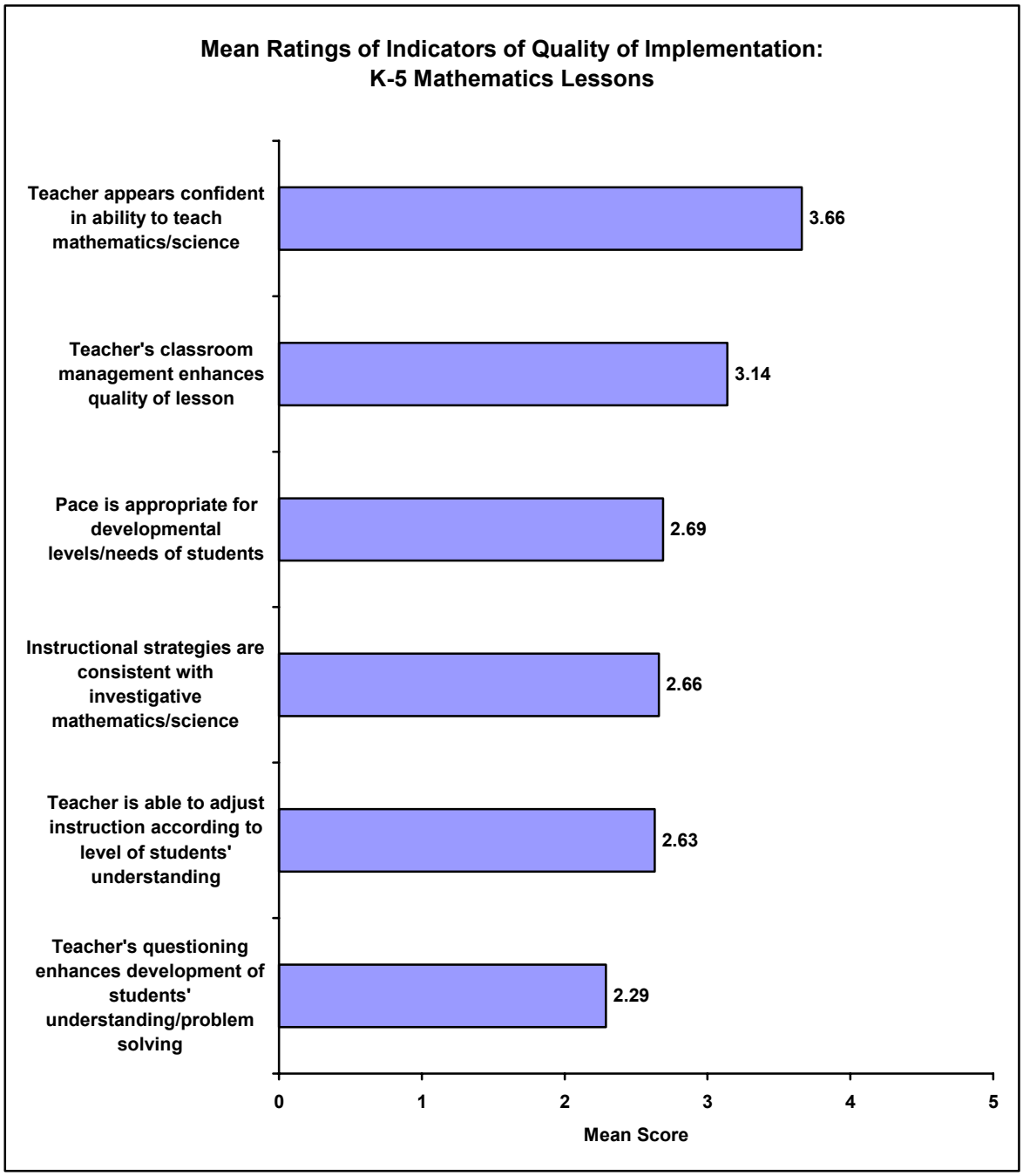
Mean Ratings of Indicators of Quality of Implementation:
K-5 Mathematics Lessons
Teacher appears confident in ability to teach mathematics/science
3.66
Teacher's classroom management enhances quality of lesson
3.14
Pace is appropriate for developmental levels/needs of students
2.69
Instructional strategies are consistent with investigative mathematics/science
2.66
Teacher is able to adjust instruction according to level of students' understanding
2.63
Teacher's questioning enhances development of students' understanding/problem solving
2.29
0
1
2
3
4
5
Mean Score

Figure C-3

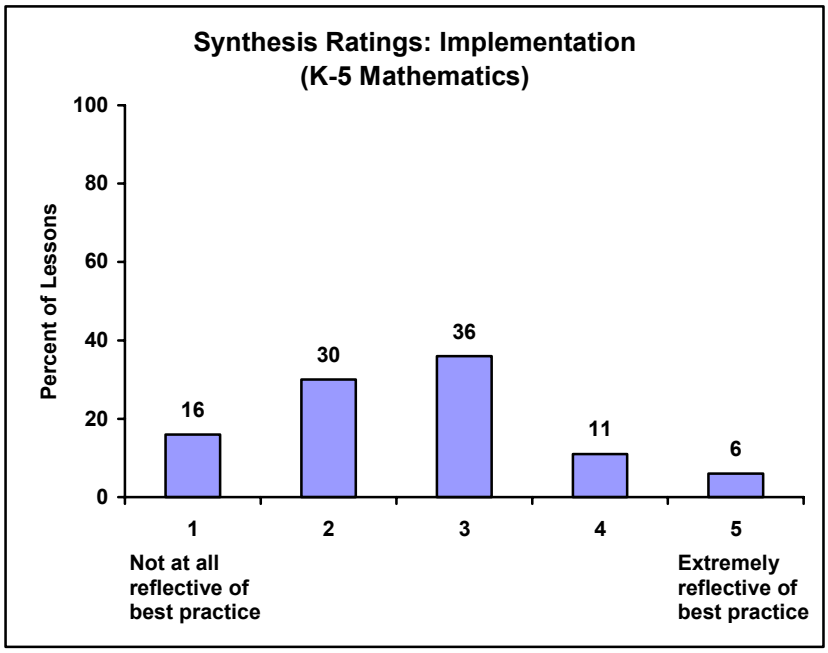

Figure C-4

The content of elementary mathematics lessons is, on average, rated highest for focusing on significant and worthwhile content and doing so accurately. Lessons are weak in engaging students with the content in a meaningful way, portraying mathematics as a dynamic body of knowledge, and providing opportunities for students to make sense of the content. Twenty-five percent of lessons receive a high synthesis rating for content, 34 percent receive a medium rating, and 40 percent receive a low rating.

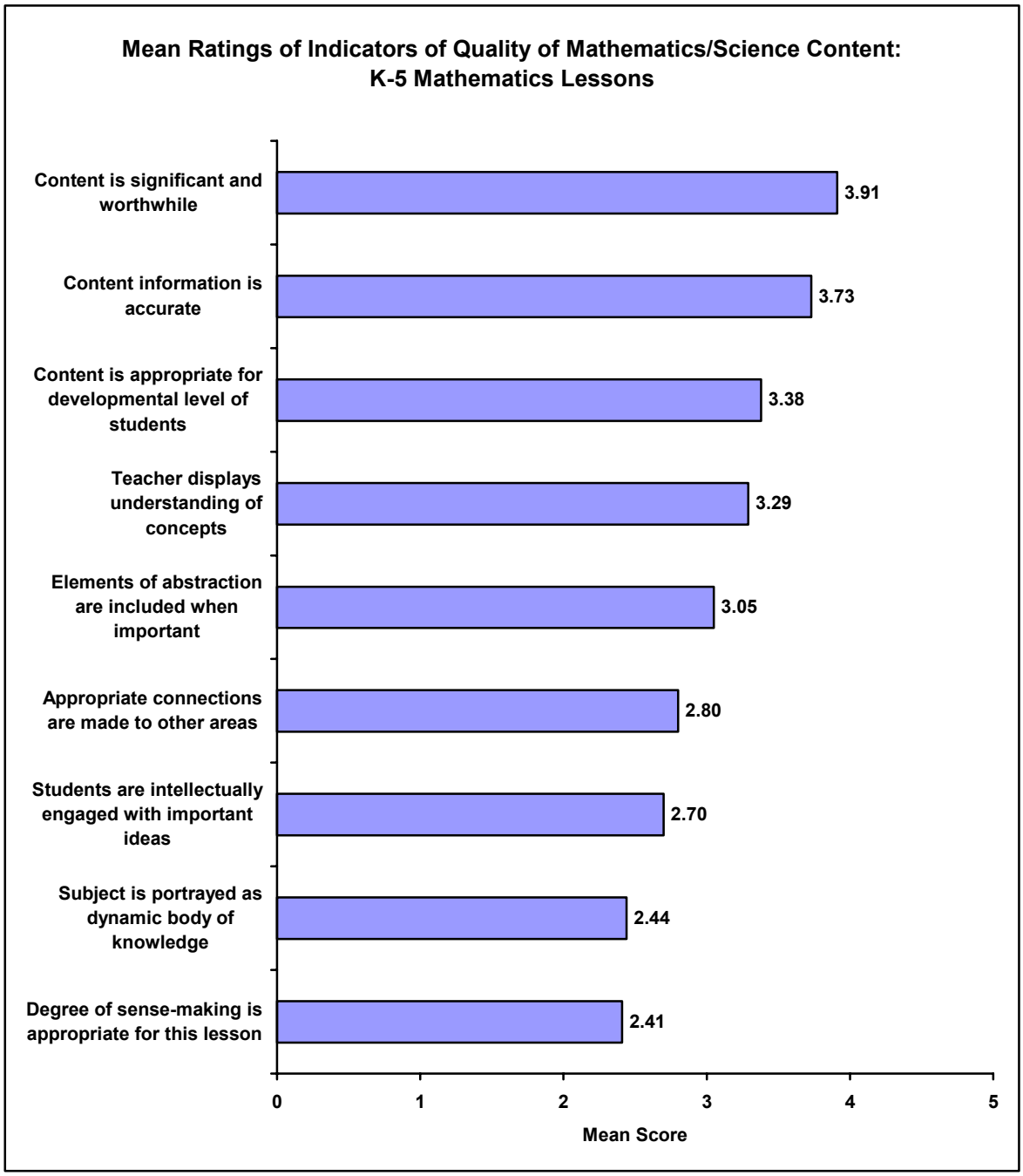
Mean Ratings of Indicators of Quality of Mathematics/Science Content:
K-5 Mathematics Lessons
Content is significant and worthwhile
3.91
Content information is accurate
3.73
Content is appropriate for developmental level of students
3.38
Teacher displays understanding of concepts
3.29
Elements of abstraction are included when important
3.05
Appropriate connections are made to other areas
2.80
Students are intellectually engaged with important ideas
2.70
Subject is portrayed as dynamic body of knowledge
2.44
Degree of sense-making is appropriate for this lesson
2.41
0
1
2
3
4
5
Mean Score

Figure C-5

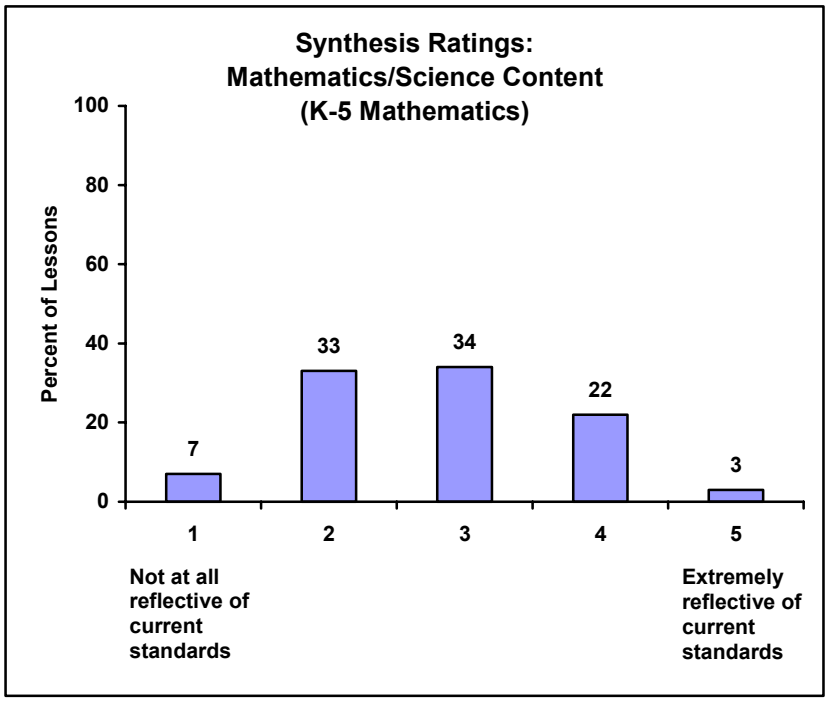

Figure C-6

The strongest aspect of classroom culture is the encouragement of active participation of all students. Although lessons are rated relatively highly for having a climate of respect for students' ideas, questions, and contributions, lessons are weak in encouraging students to generate ideas and questions. Further, lessons are weakest in their level of intellectual rigor. The synthesis ratings for classroom culture reflect these indicators with 19 percent of lessons receiving a high rating, 47 percent receiving a medium rating, and 33 percent receiving a low rating.

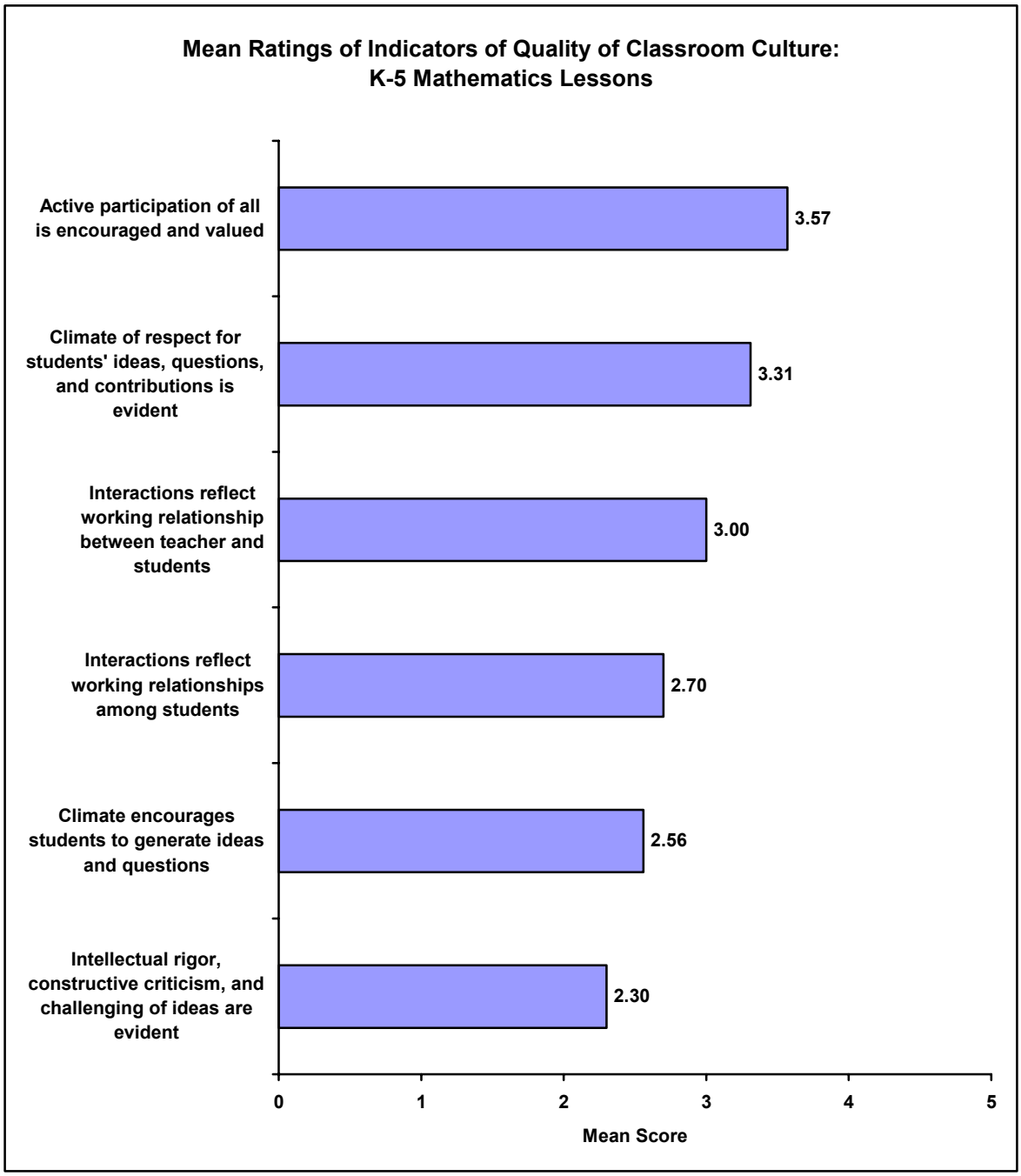
Mean Ratings of Indicators of Quality of Classroom Culture:
K-5 Mathematics Lessons
Active participation of all is encouraged and valued
3.57
Climate of respect for students' ideas, questions, and contributions is evident
3.31
Interactions reflect working relationship between teacher and students
3.00
Interactions reflect working relationships among students
2.70
Climate encourages students to generate ideas and questions
2.56
Intellectual rigor, constructive criticism, and challenging of ideas are evident
2.30
0
1
2
3
4
5
Mean Score

Figure C-7

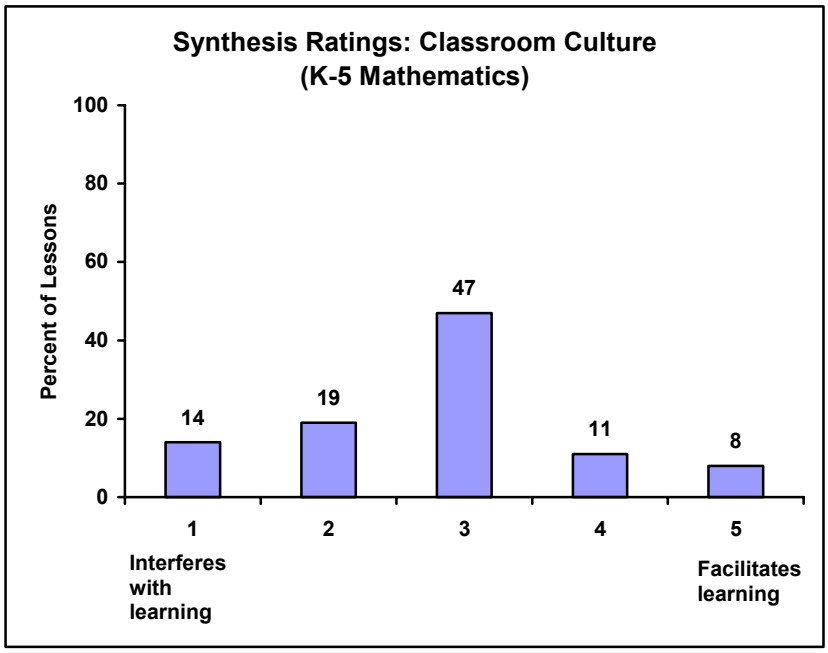
Synthesis Ratings: Classroom Culture
(K-5 Mathematics)
Percent of Lessons
100
80
60
40
20
0
14
19
47
11
8
1
2
3
4
5
Interferes with learning
Facilitates learning

Figure C-8

Overall Lesson Quality

Following the ratings of the individual components of the lesson, the researcher was asked to consider the likely impacts of the lesson as a whole. Forty-three percent of lessons or fewer have positive impacts on students' confidence to do mathematics, interest in the discipline, understanding of important mathematics concepts, or ability to apply the skills and concepts they are learning to other disciplines or real-life situations. (See Table C-1.)

Table C-1
Likely Impact of the Lessons: Mathematics K–5

	Percent of Lessons		
	Negative Effect	Mixed or Neutral Effect	Positive Effect
Students' self-confidence in doing mathematics/science	20	37	43
Students' interest in and/or appreciation for the discipline	23	34	43
Students' understanding of important mathematics/science concepts	14	48	39
Students' understanding of mathematics/science as a dynamic body of knowledge generated and enriched by investigation	22	43	35
Students' ability to apply or generalize skills and concepts to other areas of mathematics/science, other disciplines, and/or real-life situations	11	57	32
Students' capacity to carry out their own inquiries	20	49	30

Figure C-9 shows the percentage of K–5th grade mathematics lessons in the nation rated at each of a number of levels. (See page 9 of the Observation and Analytic Protocol in Appendix A for a description of these levels.) Forty-five percent of elementary mathematics lessons are rated as low in quality on the capsule rating, 37 percent are rated as medium in quality, and 18 percent are rated as high in quality.

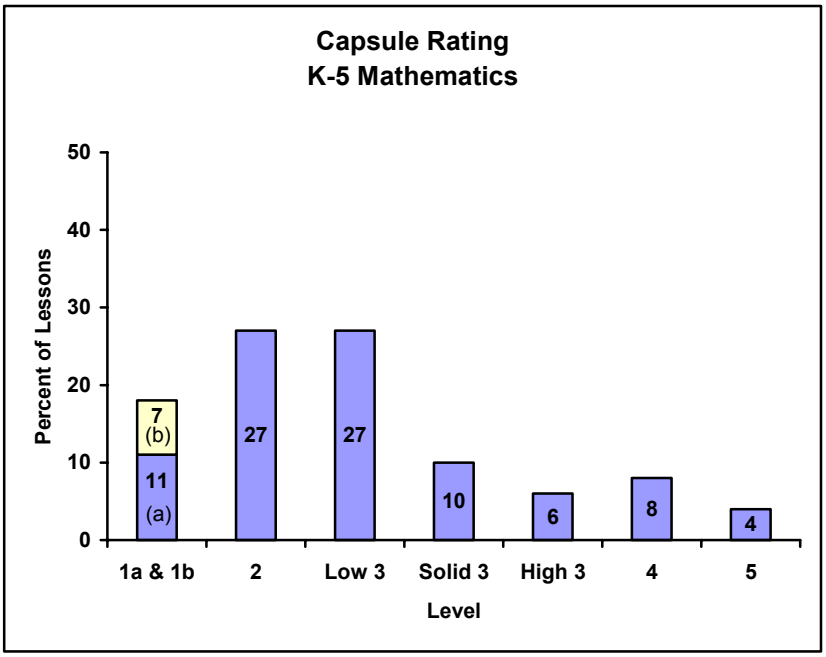

Figure C-9

The following illustrate lessons that were rated low, medium, and high in quality.

Sample Low Quality Lesson: Passive "Learning"

The teacher explained that the purpose of this 5th grade lesson was to help students better understand equivalent fractions and how to change improper fractions to mixed numbers.

The lesson started with the students participating in an oral exercise directed by the teacher. The exercise involved the teacher asking a series of questions across a very broad range of concepts. Most of the questions appeared to focus on previously learned facts, vocabulary terms, and basic operational skills, e.g., "What's the operation represented by a fraction?" and "How do we change a mixed number to an improper fraction?" Some questions asked during this part of the lesson appeared to be unrelated to the overall goal of the lesson or not well thought out. For example, the teacher asked students how to write numbers in base ten and base five. A few students volunteered answers repeatedly, while many others sat silently through the discussion.

As the teacher explained strategies to solve different problems during this oral exercise, often times his explanations were confusing. For example, one of his questions asked students to compare $^1/_5$ to $^1/_3$ and identify the sign that would separate the two quantities. In response to the teacher's question, students suggested using the less than sign and mentioned the process of cross multiplication as a means of checking to see if the symbol was correct. In following the students' thinking, the teacher proceeded to demonstrate how the process of cross multiplication could be used in this case. He multiplied 5×1 and 3×1 and wrote 5 ___ 3. The teacher then summarized the process saying, "Since 5 is greater than 3, then $^1/_5$ is less than $^1/_3$." The explanation appeared to confuse many.

Another student suggested division as a way of determining which fraction had greater value, to which the teacher responded by walking the class through the process of changing both fractions to decimals. However, upon completion of the conversions, he failed to go back and connect what this process had to do with comparing the two fractions. Instead, he launched into a discussion of how $^1/_3$ is a repeating decimal and could be written as $33^1/_3$%. The explanation went as follows: "Well, since you are dividing 100 by 3, the closest you get to 10 is 9, with a remainder of 1. So the 1 becomes $^1/_3$ or you could continue dividing and get 33.3333….You wouldn't have enough paper to carry it out, so we use the fraction form. We carry out the division to two decimal places and then use a fraction." Throughout this explanation there was nothing written on the board, nor was there anything available to help students make better sense of what was being said.

This whole group discussion lasted roughly 40 minutes with the teacher presenting information that was incomplete, confusing, or likely to foster misconceptions. Following the discussion, the teacher had students work individually on a practice worksheet for the remaining 15 minutes in the class period. Overall, the lesson did very little to help students deepen their knowledge of equivalent fractions or converting improper fractions to mixed numbers.

Sample Low Quality Lesson: "Activity for Activity's Sake"

In the prior lesson for this 4th and 5th grade class, students discussed the concept of area and were introduced to the formula of length times width. The observed lesson was designed to build on this experience by having students use manipulatives to represent area and, in the process, develop a deeper understanding of the concept.

The lesson began with a warm-up activity involving four basic computation problems written on the overhead: $98 - 39$; $80 - 59$; 8×4; 56×3. After students were given time to solve the problems individually, the teacher called on students to walk her through the answers. As correct answers were provided, the teacher wrote them on the overhead. However, her questioning strategies were limited and highly directed toward getting students to say the correct answers. Her answers to students' questions were similarly brief and did very little to help students make sense of the errors being made. Upon receiving answers to the four problems, the teacher abruptly moved to the "main lesson" for the day, without mention of how, if at all, the warm-up problems related to the topic.

The teacher started this activity with a brief recap of the prior lesson's discussion of area. She proceeded to read through the directions as they were printed on the handout, warning the students that they needed to pay attention because the directions contained several steps and were complex. The activity involved students cutting out twenty, 1 cm by 1 cm squares, coloring them, and then gluing them onto a piece of construction paper to form different rectangles. The worksheet specifically asked students to create two rectangles for each of a number of different areas, such as 8, 12, 16, 18, and 20 square centimeters.

To further orient the students to the task, the teacher demonstrated the process for an area of 6 square centimeters by forming a 1×6 and a 2×3 rectangle and hanging her paper by the overhead as an example. She then quizzed the students on the steps to take before finally giving students the opportunity to work on the activity themselves.

With roughly 20 minutes of class time remaining, the students began cutting out squares. During this time, the teacher moved about the classroom and answered questions individually. Most of these questions were about the procedures students were supposed to follow, as opposed to the concept of area. For example one student asked, "Do we color the squares now, or glue them to the paper and then color them?"

With two minutes remaining, the teacher instructed the students to clean up, and the lesson ended without any time being dedicated to making sense of the mathematics. In fact, only a handful of students actually started to address the mathematics, with others still cutting and coloring the squares.

Although understanding area is a reasonable expectation for students at this level, the teacher spent more time having students understand the steps to creating manipulatives rather than actually using the manipulatives to deepen their understanding of area. The lesson likely confused students, and helped perpetuate the myth that mathematics is about strict procedures that need to be followed.

Sample Medium Quality Lesson: Beginning Stages of Effective Instruction

The lesson in this 2nd grade class focused on getting students to visualize addition and subtraction story problems by acting them out, then drawing "cartoon" representations of them, and then representing them with a number sentence. The teacher began the lesson by telling students to look at the calendar on the bulletin board. (This was a large, brightly colored calendar with numbers and days of the week that everyone could see from a distance.) She spent the first few minutes asking rapid-fire questions about the calendar and other mathematics displays (e.g., the day of the week, the time of day, how many days until the weekend, etc.). Students eagerly answered the questions (e.g., "What is the third day of the week?" "How many more [birthdays are there] in November than in April?" "What is one less than thirty-six?"). Students went to the board as called upon to manipulate the clock to the time the teacher requested or to point to numbers as the group counted forward and backwards out loud.

Following the "math meeting," as the teacher called this portion, the lesson was introduced when the teacher reminded students of "some, some more" stories from previous mathematics lessons. She called four children by name to come forward to stand in front of the class while she said, "Four children went to the mall." She then called three more names and these students came forward as she remarked, "Three more children joined them. How many are at the mall now?" Next, students acted out a "some, some went away" story.

The teacher explained that these stories couldn't always be acted out, so they would need to draw the problem on the board or on their paper. She recited a problem from the curriculum. ("Darlene went fishing with her sister. She caught two fish in the morning. In the afternoon she caught three fish.") The teacher drew simple fish shapes on the board and asked the children to classify this problem ("some, some more" as opposed to "some, some went away"). And she asked a volunteer for a number sentence to describe the problem.

In the next three examples, students came forward to draw the objects in the word problems (pencils, chickens, eggs, etc.). Other students were asked to supply number sentences. When a student supplied the wrong answer, the teacher questioned further (e.g., "A minus sign? Are we taking away?" or "Is there a three up there? You need to pay attention.").

With no real wrap-up, the students were next given a worksheet containing 25 addition problems to test their proficiency with "plus-two" facts. The students were clearly familiar with this drill of doing as many problems as they could in one minute. They began working excitedly when the teacher said, "On your mark, get set, go." After one minute, children pulled out colored crayons to correct their work. The teacher read each problem and called out the answer while students marked their papers. She then asked for a show of hands to see how many students scored more than 15 correct, and then who had all 25 correct. The last part of the lesson was spent with the children completing a worksheet. The reverse side of the worksheet contained similar problems that were to serve as homework for the evening. The lesson ended, and the teacher instructed students to get out their language notebooks.

The teacher emphasized drill and repetition in the use of the mathematics meeting and the worksheets. Connections to daily life were obvious during the "math meeting" as they talked about the calendar and in the story problems that students did at the board. The teacher made it very clear that she expected the participation of all and made a point of calling on nearly every child at least once during the lesson. She seemed to call on them with tasks that they could do, for the most part. Though she kept them "on their toes" with rapid questioning, the questions were nearly all directed towards getting the right answer, rather than sharing their thinking, and she sometimes short-circuited by letting another student supply an answer instead of letting the first student grapple with the mathematics.

Sample High Quality Lesson: Traditional Instruction

The goal of this 2nd grade lesson was to strengthen students' ability to count money. Students had already learned how to count money using pennies, nickels, and dimes; today's lesson focused on counting quarters.

The lesson began with a 20-minute review where students were seated on the carpet in an area of the classroom away from their desks. During this time, the teacher reviewed a broad variety of skills, including some that had connections to science, such as reading a thermometer and completing a weather graph. The teacher used a variety of visual aids such as real money, color tiles, and erasable pens for marking on a bulletin board graph.

The variety of activities maintained student interest. Questions like, "What are some different ways to make $0.66 with coins?" encouraged a number of different student responses. In addition, questions like "What are some different number sentences that will make 23?" encouraged students to invent number sentences such as "23 × 1," "23 + 0," and "10 + 13." Students had many opportunities to write on the bulletin board, write on the white board, and manipulate money and color tiles. At the close of the review, the teacher had students participate in a skip counting exercise as they returned to their desks.

The teacher then began to present the new material on counting quarters, asking students to compare different numbers of quarters to one dollar, to count by 25, to tell how much 7, 5, or 9 quarters equals, and to tell how many quarters were in different dollar amounts, such as $2.25. Symbols and drawings were effectively used to connect these oral responses, and the teacher's questions did a fine job of challenging students' thinking. Questions included: "What is ¾ of a dollar?" and "How many quarters make $1.25?" The teacher also used tiles to aid students' counting and a drawing to relate fourths to quarters in a dollar.

Approximately five minutes were spent at the end of the lesson on a subtraction fact review. Pairs of children reviewed basic subtraction facts at different places in the room while the teacher walked around to check their progress.

The teacher used the script and materials provided by the textbook to implement this well-paced and well-organized lesson on counting money. The teacher guided the entire lesson, so there were no opportunities for student-initiated investigation. However, the variety of instructional strategies provided many opportunities for students to explore the concept of counting money in different ways, and the teacher built in adequate time for processing and sense making. Throughout the lesson the teacher maintained focus on the content goals and incorporated challenging questions that maintained intellectual rigor. The lesson was highly likely to strengthen students' ability to count money.

Sample High Quality Lesson: Reform-Oriented Instruction

This 3rd grade class had recently finished studying congruency and symmetry. This lesson focused on coordinate geometry in the first quadrant. The teacher stated that the purpose of the observed lesson was to get students to be able to read a graph using coordinate pairs and to know how to find a specific point in the first quadrant.

The teacher introduced the topic for the day by relating the content of the coordinate system to students' prior work with mapping and directions. To help make the comparison more real, she asked all the students to close their eyes and began to talk through an example to show how following specific directions leads to an exact spot. She stated, "Go out this door. Turn right. Go through the double set of doors. Go a few feet further. Whose room is to the right?"

The class in unison yelled the name of the teacher who teaches in that room. She gave another similar example, and students once again in unison called out the location. She then smoothly segued into how coordinate pairs are similar but much more specific. She made another comparison to city streets and walked the class through the process of setting up the first quadrant of the coordinate plane. During this time she wove in important vocabulary terms such as quadrant, vertical, and horizontal. She also allowed students the opportunity to challenge one another's answers by asking questions such as "Is this correct?" and "Does anyone have a different idea?"

After students practiced to create their own grid, the teacher then introduced a ready-made grid for the overhead and gave each group a laminated grid of the first quadrant for the upcoming assignment. However, neither grid had the axes numbered, so she walked the class through how to number the grid for these particular examples.

The teacher then guided the class through a couple of examples. She called students up to do other examples, and then let the students work in pairs on a few more. Instead of placing a regular point on the position, she had cut out pictures of various objects such as balloons, cars, and people for students to place at the spot. As pairs worked on plotting their coordinates, the teacher walked around and monitored their work. After pairs completed the assignment, the teacher called the class together to go over their work. Students came up and placed pictures on the overhead version of the grid, and the teacher allowed other pairs to comment on the correctness of the placement.

This sense-making activity was followed by a worksheet for students to complete individually. The worksheet had similar questions but also extended their knowledge by asking them to list the coordinates instead of finding the coordinates. Although each student had an individual sheet to finish, the entire table of four students was responsible for helping those that needed more guidance. The teacher ended the lesson by going over the worksheet and discussing common errors.

The lesson had a very strong design that attended to a variety of learning styles and excellent implementation including questioning that clearly moved students forward. The lesson was highly likely to improve the students' understanding of coordinate pairs.